Computer Simulation of Pressure-Flow in Aorta and Aortic Coarctation

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Abstract

With the method of characteristics, pulsatile pressure and velocity pulses are computed in a tapered and non-branching distensible tube simulating the aorta. The elastic modulus of the tube is assumed to vary spatially and with pressure. The relationship of pressure and diameter is an exponential function similar to the observed tension-length diagram. The model is also applied to a constricted tube simulating coarctation of aorta. Calculations show an alteration of pressure and velocity pulses distal to the stenosis.

One of the distinctive characteristics of propagation in the aorta is the change in shape of the pressure and flow pulses as they move from the heart to the periphery of the vascular system. Several models (1,2,3,4,5) have been proposed for the pressure flow relationship, yet in these models the elastic modulus is considered pressure independent. It is the purpose of this paper to describe a model in which the Young's modulus is not only a function of pressure but also varies spatially. The pressure-flow patterns are presented in both normal aorta and aortic coarctation with different degree of constriction.

The basic assumptions needed to develop the mathematical model for claculation of pressure and velocity are:

1. One dimension flow. Flow is given by average velocity over the cross section.
2. The ratio of thickness of the tube wall to diameter of the tube in unstressed state (ho/Do) =0.1.
3. The wall material is incompressible. This implies a Poisson's ratio of 0.5.
4. The diameter of the tube is given by

   $$D(x)=D_a-\frac{D_a-D_f}{L}x-\alpha D(x_a)\exp(-B(x-x_a)^2)$$

   in which D_a, D_f are the diameters of the tube at x=0 and x=L
   D (xa) is the diameter at x=xa and α is a constriction factor.
5. Young's modulus E (x.t) varies with pressure and distance. i.e.

   $$E(x,t)=KP(x,t)+E_o(x)$$

   where E (x) is the elastic modulus during unstressed state. k is constant of proportionality.
6. Shear stress at the tube wall due to flow is proportional to V^2 and is opposite to the direction of flow.
7. At time=0, an initial flow entering at n=0 and a pressure difference of 0.5 mmHg. at x=L are assumed.

Also from the equation of continuity and momentum equation for a segment of the tube: the following equations are obtained.

$$\frac{P_x}{\rho}+VV_x+V_t+\frac{FV^2}{2D}=0$$

$$V_x+\frac{A_{ox}}{A_o}V+V\frac{P_x}{a^2}+\frac{P_t}{a^2}+V\frac{E_{ox}}{E_o}\frac{P}{a^2}=0$$

$$a^2=Eh/D$$

The above two equations comprise a system of two nonlinear hyperbolic partial differential equations in two dependent variables, P and V. These equations may be solved numerically using the method of characteristics.

Fig. (1) shows the input pressure at x=0 and the computed pressure at x=20 cm. and 40 cm. The peak value of the pressure pulse increases and the leading edge of the pulse becomes steeper as it propagated from the proximal to the peripheral part of the arterial system.

The shape of the pulse also becomes narrower. The sharp incisura disappears and the dicrotic notch and a dicrotic wave appears. In contrast, the amplitude of the velocity pulse (Fig. 2) decreases and its shape becomes somewhat broader. Its leading edge becomes less steep. The backflow component also becomes smaller.

The progressive change of pressure and velocity pulse immediately distal (5 cm. below the constriction) to the constricted region are shown in Fig. (3) and Fig. (4). These pressure pulses are computed in a constricted tube without dilatation. The curves exhibit a progressive reduction in amplitude and a retardation of anacrotic limb. The pulse contour, instead of being narrow, is broad and tends to be rounded.

With moderate constriction, the systolic pressure no longer exceeds aortic pressure as is normally the case, it either equals it (at 36% reduction) or is less with further constriction. The dicrotic wave becomes less and less prominent. Similar to the pressure pulse, the velocity amplitude diminishes with progressive degree of constriction. The controur tends to be broadened with reduction of backflow component.

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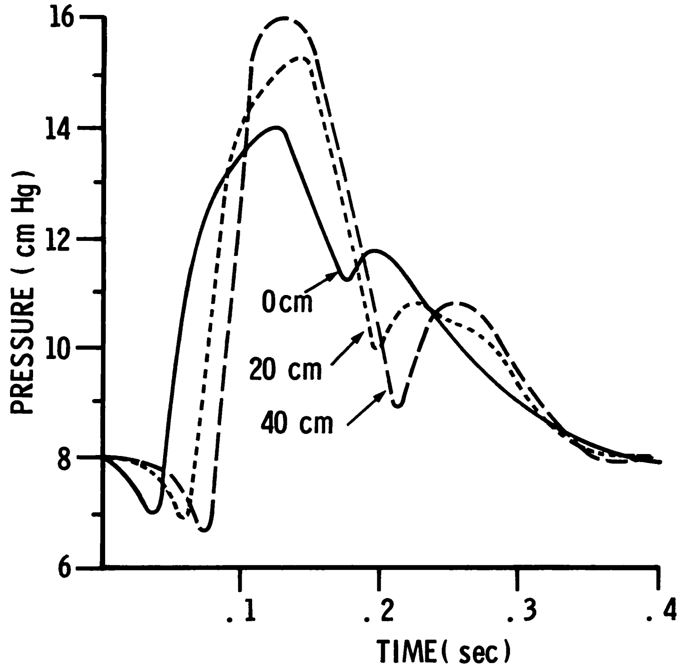

Fig. (1) Input pressure and the computed pressures as function of time at 3 locations down the model aorta.

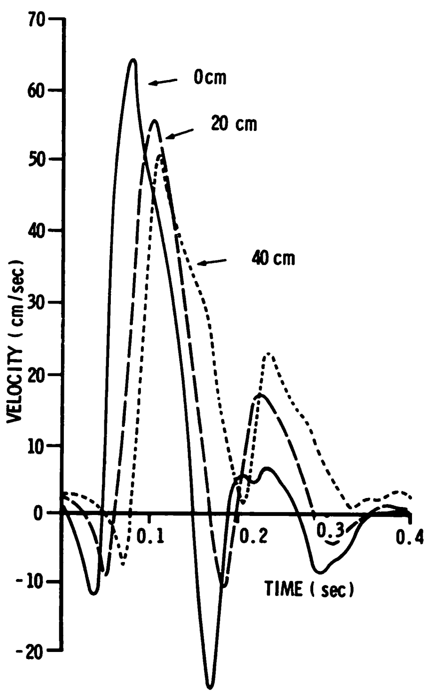

Fig. (2) Computed velocity as function of time at 3 locations as indicated.

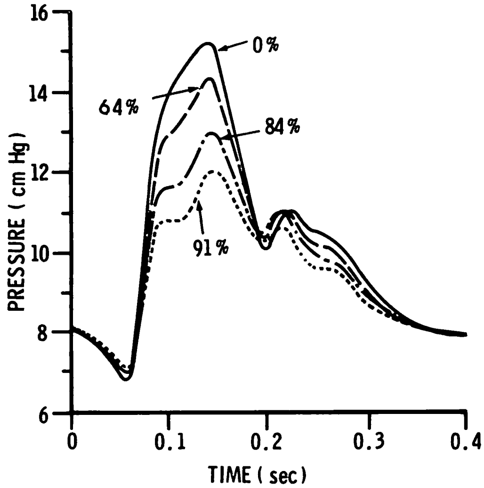

Fig. (3) The computed pressure pulses at 20 cm. station. The percentage indicates reduction of cross-section area at 15 cm. station.

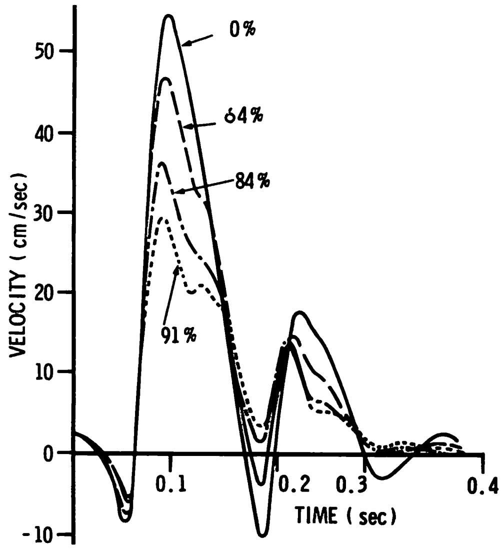

Fig. (4) The computed velocity pulse at 20 cm. station due to various degrees of constriction.

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