ELASTIC EXPULSION

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ABSTRACT

The paper is a mathematical and experimental investigation of the discharge of fluids from long, distensible tubes following instantaneous severance. With reference to myelinated nerve fibres, laticifers of trees and the mammalian venous system, the mathematical study considers flow of a power-law fluid from a long, uniform, initially-distended tube. Similarity solutions are developed from which pressure, velocity, distension and exudation rate are determined.

Discussion of experiments includes a review of earlier work on myelinated nerve fibres and the results of recent experiments using rubber tubes and the canine venous system. The experimental results are compared with the mathematical predictions.

INTRODUCTION

The work presented here arose from an earlier study [1] devoted to the measurement of axoplasm exudation rates from myelinated nerve fibres. In [1] it was concluded that the flow was driven by virtue of contraction of the initially-distended myelin "tube" following severance: that is, the axoplasm underwent elastic expulsion. This conclusion is in agreement with the suggestion of Lubinska [2] who, in turn, referred to an earlier study of latex flow by Frey-Wyssling [3].

In view of the widespread importance of long tubes in the structure and function of animals and plants, as exemplified by the work discussed above, it appeared worthwhile attempting a discussion of elastic expulsion in more general terms. Accordingly, a mathematical analysis and a series of physical and physiological experiments were undertaken.

MATHEMATICAL ANALYSIS

The first attempt to discuss elastic expulsion rigorously appears to be that of Hermans [4]. Although the approximations implicit in [4] are not wholly justified it happens that they are correct for the latex flow problem considered. Hermans' analysis is limited because the fluid is considered to be Newtonian, the tube wall linearly elastic and the amount of distension taken to be vanishingly small.

Considering the problem more generally [5] it is possible to derive a partial differential equation which describes the variation of the tube wall inner radius R(X, t) with time (t) and axial distance (X) from the point at which the tube is severed. This equation takes the form

$$\frac{\partial R}{\partial t} = A\frac{\partial^2 R}{\partial X^2} + B\left(\frac{\partial R}{\partial X}\right)^2 + C\frac{\partial R}{\partial X} \qquad (1)$$

where the coefficients A, B & C are dependent upon X and t but also functions of the rheology of both the fluid and the tube wall. It is clear that the complexity of the coefficients A, B & C is a measure of the complexity of the problem.

In the work of Hermans [4], A = 1 and B = C = 0 and hence the above differential equation reduced to the relatively simply one-dimensional diffusion equation. If, however, the fluid obeys a power law relation between shear stress and rate of shear strain, and if the expression relating fluid pressure (P) to tube inner radius takes either of the forms:

$$P - P_o = K_p[1 - e^{-m_p(R - R_o)/R_o}] \qquad (2)$$

$$R - R_o = K_R[1 - e^{-m_R(P - P_o)/P_o}] \qquad (3)$$

then none of the coefficients takes on a simple form. Even so, equation (1) may still be solved by converting it to an ordinary differential equation in which

$$\phi(\eta) = \frac{R - R_o}{R_1 - R_o}{}^{*},$$

is dependent upon the similarity variable η and three parameters: R_o/R_1, reflecting the magnitude of the dilatation; n, the index of the power-law fluid and m, measuring the departure of the elastic wall from linearity. The independent variable η incorporates not only the variables X and t but rheological constants also.

Solutions to this problem have been found by iteration from simply zeroth solutions [5] and reveal the effects of the above-mentioned parameters [6]. The solutions assume that the fluid is initially at rest and the tube is severed instantaneously.

EXPERIMENTAL WORK

A series of experiments with rubber tubes

*Subscripts 1 and o refer to initial and final conditions, respectively.

was undertaken to ascertain the range of validity of the theoretical predictions. For convenience, a Newtonian fluid (glycerine-water solution) was chosen: viscosity could be varied over about one order-of-magnitude. The rubber tubes used were found to obey a relation of the form (2) under quasi-static calibration conditions ($m_R \simeq 3$): it must be acknowledged, however, that their dynamic behaviour may involve a time dependence not incorporated in equation (2).

The apparatus was arranged as shown in Figure 1 and the experiments, which were designed to monitor fluid pressure, were executed as follows. With the tube initially pressurized by means of a header tank the strip-chart recorder was started. After waiting a short period to ensure that conditions had stabilized, the tube was severed "instantaneously" by means of a guillotine, the descent of which was used to produce a timing mark on the strip-chart record at the "moment" of severance. The pressure record thus obtained was then used in combination with equation (2) to determine $R(X, t)$ and hence $\phi(\eta)$. The experimental results were then compared with the theoretical predictions, revealing good agreement.

In vivo experiments were also conducted on the canine venous system, within which the conditions assumed in the mathematical theory should hold: that is, the vessels should be long compared with their diameter and the pressure should be non-pulsatile. For veins it appears that equation (3) is a more realistic description of the pressure-radius relation than equation (2).

Three small (5Kg) heparinized dogs were used in a series of experiments on the femoral, cephalic, saphenous and mesenteric veins: severance was effected with fine scissors. The results of these physiological experiments were then compared with the results of the physical experiments and the theoretical analysis.

CONCLUDING REMARKS

Comparison of theoretical and experimental results reveals good agreement under the conditions assumed: this is to be expected on the basis of an order-of-magnitude analysis. The results also indicate the significance of neglecting axial restraint of the tube and suggest that flexural rigidity may not be negligible near the point of severance. These factors, together with the dynamic aspects of tube wall rheology deserve further study.

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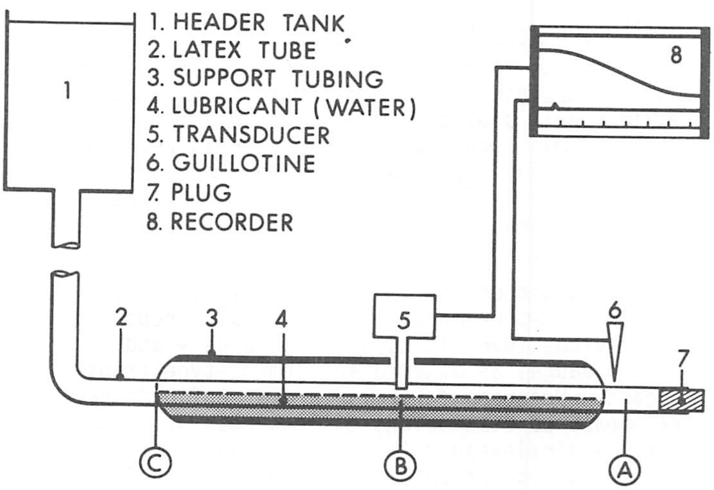

Figure 1 Schematic of Apparatus