ITERATIVE BLIND SOURCE SEPARATION OF EMG SIGNALS BASED ON THE JOINT DOMAIN SPARSITY

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INTRODUCTION

Electromyogram (EMG) signals are composed of a mixture of motor unit action potential (MUAP) trains where the observed EMG signal at each sensor can be modeled as a convolutive mixture of the motor unit (MU) signals. EMG signal decomposition identifies and classifies each individual MUAP. Decomposition of EMG signals is important for detecting physiological abnormalities, analyzing the biomechanics of muscle movement, and to study MU recruitment order and patterns [1].

The number of source MUs is typically larger than the number of EMG sensor channels. Since the number of sources is greater than the number of sensors, and the convolutive mixing matrix is unknown, the decomposition problem can be considered as an underdetermined blind source separation [2][3][4].

One important observation is that MUAPs are only occasional from a MU. Therefore, EMG signals are composed of signals that are sparse in the time domain. In addition, the frequency content of MUAPs is not dense and hence EMG signals are also sparse in the frequency domain. Using this joint domain sparsity, a MUAP train could be estimated with fewer sensors than number of MUs from the mixture EMG signal.

The main objective of this paper is to develop an algorithm which takes advantage of the sparsity inherent in EMG signals to estimate the MU signals with as few sensors as possible, such that individual MUAP can then be identified. In this paper, a new optimization problem is presented based on the joint time and frequency domain sparsity of EMG signals for this signal separation.

SYSTEM MODEL

Assume that we have N different MU signals. Further assume that $x_n(t)$ is the signal corresponding to the n^{th} MU, where t is the time index and $n = 1, \ldots, N$. Measuring the N source MU signals through M electrodes, each recorded signal $y_m(t)$ can be modeled as the filtered and mixed signal

$$y_m(t) = \sum_{n=1}^{N} \sum_{l=0}^{L} a_{mn}(l) x_n(t-l) \tag{1}$$

where $a_{mn}(l)$, $l = 0, \ldots, L$, are FIR filter coefficients between the n^{th} source signal and the m^{th} electrode and $L + 1$ is the filter length.

Assume that $T + L$ is the length of the time slot for the n^{th} source signal $\bar{\mathbf{x}}_n$ and T is the length of the mixture signal $\bar{\mathbf{y}}_m$ at the m^{th} sensor. The vectors $\bar{\mathbf{x}}_n$ and $\bar{\mathbf{y}}_m$ are defined, respectively, as

$$\bar{\mathbf{x}}_n = [x_n(T+L), x_n(T+L-1), \ldots, x_n(1)]^{\mathrm{T}} \tag{2}$$

$$\bar{\mathbf{y}}_m = [y_m(T), y_m(T-1), \ldots, y_m(1)]^{\mathrm{T}}. \tag{3}$$

Now, (1) can be written as

$$\bar{\mathbf{y}} = \mathbf{A}\bar{\mathbf{x}} \tag{4}$$

where the extended source vector $\bar{\mathbf{x}} = [\bar{\mathbf{x}}_1, \ldots, \bar{\mathbf{x}}_N]^{\mathrm{T}}$ is the concatenation of the $(T+L)\times 1$ vectors $\bar{\mathbf{x}}_n$, for $n = 1, \ldots, N$ and the extended observation vector $\bar{\mathbf{y}} = [\bar{\mathbf{y}}_1, \ldots, \bar{\mathbf{y}}_M]^{\mathrm{T}}$ is the concatenation of the $T \times 1$ vectors $\bar{\mathbf{y}}_m$, for $m = 1, \ldots, M$.

The mixing matrix $\mathbf{A}$ is defined as

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_{11} & \cdots & \mathbf{A}_{1N} \\ \vdots & \ddots & \vdots \\ \mathbf{A}_{M1} & \cdots & \mathbf{A}_{MN} \end{bmatrix} \tag{5}$$

where

$$\mathbf{A}_{mn} = \begin{bmatrix} a_{mn}(0) & \cdots & a_{mn}(L) & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & a_{mn}(0) & \cdots & a_{mn}(L) \end{bmatrix} \tag{6}$$

is a $T \times (T+L)$ block Toeplitz matrix, $\forall m,n$, which can be proved to have the restricted isometry property (RIP) [5].

TWO-STEP SEPARATION ALGORITHM

The EMG decomposition problem is to separate the N source MU signals from the M observations, while the mixing matrix $\mathbf{A}$ is unknown. Here, we propose an iterative algorithm with two steps in each iteration. In the first step, estimates are made of all N source signals from the M observations. In the second step, the mixing matrix $\mathbf{A}$ is estimated. This process is repeated to refine the estimates of both the source signals and mixing matrix.

Motor unit source signal estimation

Since the source signals are jointly sparse in the time-frequency domain, it is advantageous to use wavelet domain representation of the source signals. (4) can be rewritten in the time-frequency domain as follows

$$\bar{\mathbf{y}} = \mathbf{A}\bar{\mathbf{x}} = \mathbf{A}\mathbf{\Phi}\bar{\mathbf{s}} \tag{7}$$

where $\bar{\mathbf{s}}$ is the sparse time-frequency representation of $\bar{\mathbf{x}}$. The concatenated orthonormal basis $\mathbf{\Phi}$ is defined as

$$\mathbf{\Phi} = \begin{bmatrix} \mathbf{\Phi}_{11} & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{\Phi}_{NN} \end{bmatrix} \tag{8}$$

where $\mathbf{\Phi}_{ii}$ is a $(T+L)\times(T+L)$ orthonormal basis that is the same for all i.

As outlined previously, $\bar{\mathbf{x}}$ and $\bar{\mathbf{s}}$ are *a priori* known to be sparse with a small number of nonzero components while the rest of the components are very small and considered to be zero. Under this sparsity assumption, $\bar{\mathbf{s}}$ can be estimated with fewer sensors as compared to the number of sources.

Based on the joint time-frequency sparsity of the source signals, the following optimization constraint is introduced

$$\begin{aligned} \min_{\bar{\mathbf{x}},\bar{\mathbf{s}}} \quad & \lambda\|\bar{\mathbf{x}}\|_1 + (1-\lambda)\|\bar{\mathbf{s}}\|_1 \\ \text{s.t.} \quad & \bar{\mathbf{y}} = \mathbf{A}\bar{\mathbf{x}} \\ & \bar{\mathbf{y}} = \mathbf{A}\mathbf{\Phi}\bar{\mathbf{s}} \end{aligned} \tag{9}$$

where $\|\bar{\mathbf{f}}\|_1 = \sum_{i=1}^{N(T+L)} |\bar{f}_i|$ is the ℓ_1-norm of $\bar{\mathbf{f}}$ and λ is a weighting factor. The problem in (9) is a convex problem and can be solved using convex programming software packages such as CVX [6].

Mixing matrix estimation

At initialization, we can randomly set the elements of the mixing matrix $\mathbf{A}$, with the constraint that the block Toeplitz structure introduced in (5) and (6) is maintained. Then during the estimation of the mixing matrix, we have a minimum mean square error optimization, with the constraint that $\mathbf{A}$ has orthogonal columns. This constrained optimization problem can be written as the minimization of the Lagrangian [7]

$$\mathcal{L}(\mathbf{A},\boldsymbol{\mu}) = \|\bar{\mathbf{y}} - \mathbf{A}\bar{\mathbf{x}}\|_2^2 + \mathrm{Tr}\left[\boldsymbol{\mu}\left(\mathbf{A}^\top\mathbf{A} - \mathbf{I}\right)\right] \tag{10}$$

where $\boldsymbol{\mu}$ is an $(L+T)N \times (L+T)N$ matrix of Lagrange multipliers and $\mathbf{I}$ is the identity matrix. By setting the gradients $\nabla_{\boldsymbol{\mu}}\mathcal{L}$ and $\nabla_{\mathbf{A}}\mathcal{L}$ to zero, the mixing matrix can be computed using

$$\begin{aligned} \bar{\mathbf{y}}\bar{\mathbf{x}}^\top &= \mathbf{U}\mathbf{D}\mathbf{V}^\top \\ \mathbf{A} &= \mathbf{U}\Sigma\mathbf{V}^\top \end{aligned} \tag{11}$$

where $\mathbf{UDV}^{\mathsf{T}}$ is the singular value decomposition (SVD) of $\overline{\mathbf{y}}\overline{\mathbf{x}}^{\mathsf{T}}$ and Σ is an identity matrix.

<u>Two-step blind source separation algorithm</u>

The two-step blind source separation algorithm framework is given in Algorithm 1:

Algorithm 1: Two-step blind source separation

Inputs: $\overline{\mathbf{y}}$

Outputs: $\mathbf{A}$, $\overline{\mathbf{x}}$

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1. Initialize mixing matrix $\mathbf{A}$ randomly based on the structure introduced in (5) and (6)
2. Estimate $\overline{\mathbf{x}}$ by solving (9)
3. Compute the singular value decomposition $\overline{\mathbf{y}}\overline{\mathbf{x}}^{\mathsf{T}} = \mathbf{UDV}^{\mathsf{T}}$
4. Update $\mathbf{A} = \mathbf{U}\Sigma\mathbf{V}^{\mathsf{T}}$
5. If the stopping criterion is not reached, go to step 2.

The stopping criterion selected for this paper is the ℓ_2-norm of difference between successive estimates of $\overline{\mathbf{x}}$ (*i.e.*, $\left\|\hat{\mathbf{x}}^{(j)} - \hat{\mathbf{x}}^{(j-1)}\right\|_2^2$, where $\hat{\mathbf{x}}^{(j)}$ is the source signal estimate at the j^{th} iteration).

SIMULATION RESULT

To obtain an initial understanding of how the proposed algorithm will perform under a controlled testing, we have used simulated EMG (sEMG) locust signals where mixing parameters can be investigated. Note that this study is still preliminary and not yet a comprehensive evaluation of performance at this time, but will demonstrate functionality of the approach. Since locust EMG signals typically have no more than two or three simultaneously active MUs and each muscle has up to five MUs, we simulated the signals with $N = 5$ MUs with random muscle innervations sites and then generated MUAPs from each MU at uniformly randomized times. For the observed EMG signals in this underdetermined problem, $M = 3$ convolutive mixtures of the five MUs were simulated where the channel between each MU source and observation electrode is modeled by an FIR filter. In our simulation, the MUAPs come from five real locust EMG signals with just one active MU recorded. These real locust EMG signals were recorded with needle extracellular electrodes inserted in the forewing first basalar muscle (M97).

To generate the mixture signal of sensor m, the convolutive mixing matrices $\mathbf{A}_{mn}$, $n = 1, \ldots, N$ should be modeled. As can be seen in (6) this matrix is completely determined by $L + 1$ FIR filter coefficients for each of the M filters. In our simulation, the three mixture signals are generated with $L = 9$ with the filter coefficients selected from a normal distribution. The mixture signals of all three sensors are shown in Fig. 1. We further assume that the observation signals are corrupted by additive white Gaussian noise with SNR=10 dB.

During the separation process, the orthonormal basis $\mathbf{\Phi}_{ii}$, $i = 1, \ldots, N$, used corresponds to the Daubechies wavelet with four coefficients. The results for applying the approach on the SEMG mixture signals from Fig. 1 are shown for sources #3 and #5 in Fig. 2 and Fig. 3, respectively, with sources #1, #2, and #4 showing similar results but excluded due to space limitation. From Fig. 2 and Fig. 3, we see that reasonable visual similarity exists between the original source signal and the reconstructed signal, showing good performance for the source separation.

As an objective measure of performance, we computed the normalized root mean square error (RMSE) for the source signal versus separated (reconstructed) signal using

$$RMSE = \frac{1}{N}\sum_{i=1}^{N}\frac{\left\|\hat{\mathbf{x}}_i - \overline{\mathbf{x}}_i\right\|_2}{\left\|\overline{\mathbf{x}}_i\right\|_2}. \tag{12}$$

Averaged over 100 runs, the computed RMSE was 0.0409, which again indicates good performance in the source separation.

CONCLUSIONS

In this paper, we proposed an underdetermined blind source separation algorithm for EMG signal decomposition that takes advantage of EMG signal sparsity. Given our choice of N = 5 MUs and M = 3 observations, good performance in EMG signal decomposition is achieved, which would be suitable for simple muscles with few MUs as for the M97 muscle of a locust.

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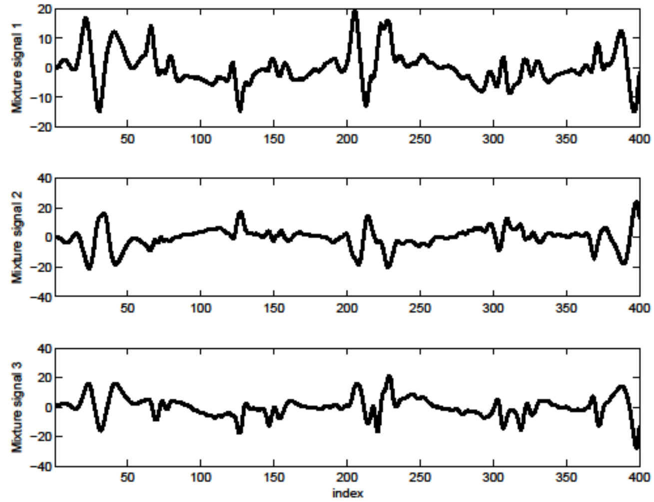

Figure 1: $M = 3$ simulated observation signals generated based on $N = 5$ MUs and a randomly generated mixing matrix $\mathbf{A}$ from (6) with $L = 9$.

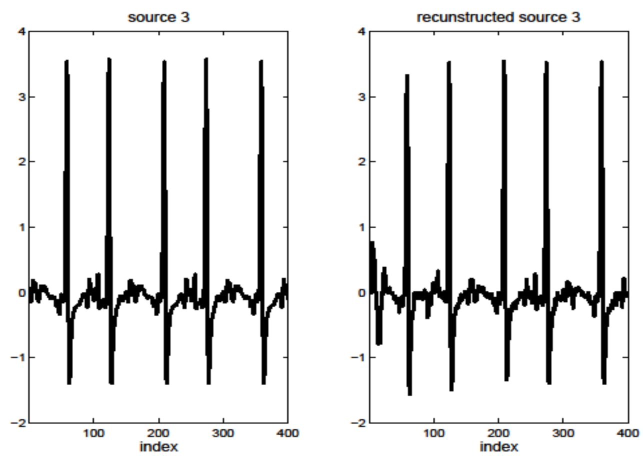

Figure 2: Original signal $x_3(t)$ for MU #3 (left) and corresponding reconstruction (right).

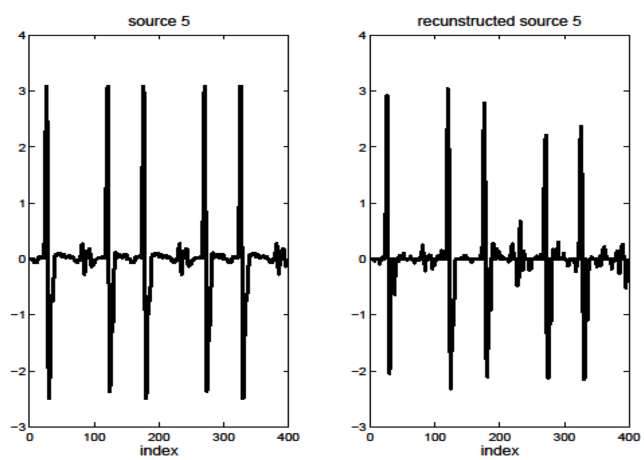

Figure 3: Original signal $x_5(t)$ for MU #5 (left) and corresponding reconstruction (right).