# PHYSICAL MEASUREMENT OF LOUDNESS

Elad Sagi and Kenneth H. Norwich
Institute of Biomaterials and Biomedical Engineering
and
Department of Physiology
University of Toronto

## INTRODUCTION

*Loudness* refers to the subjective experience associated with the intensity (mean square pressure) of a sound wave. There is a non-linear relationship between intensity and loudness. The *objective appraisal of loudness* would, then, seem almost an oxymoron. However, it is probably no more of an oxymoron than the term *functional imaging* of the brain. The functional image can be an objective, electromagnetic measure of what is otherwise a subjective, cerebrally mediated process. The loudness of a pure tone at a fixed frequency can, by hypothesis, be measured by the magnitude of the error made by the listener in identifying this tone. That is, by hypothesis,

*Loudness = Error in Identification*

## ERRORS IN IDENTIFICATION

Human participants were trained to identify to the nearest decibel tones that lay in the range *1* to *R* dB. They will sometimes succeed, but more often make mistakes. It has been observed that the errors made by participants are very nearly distributed in accordance with the normal probability density. That is, for any tone of intensity *T*, where $1 \le T \le R$, we find that the distribution of identifications, *X*, is given by

$$p(X) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(X-T)^2}{2\sigma^2}\right)$$

The participant's response, *X*, is, of course, limited by the finite range $1 \le X \le R$, so that the responses cannot be perfectly normal. However, these *anchor effects* can be corrected (Sagi and Norwich, 2002) and we shall not elaborate here. The truly extraordinary finding is that $\sigma^2$, the variance of the normal distribution, has the same value for all tones, *T*, in the range $1 \le T \le R$. For example, if the range extends from *1* to *50* dB, a tone of *32* dB will generate a "bell" curve with the same variance as a tone of *48* dB. However, $\sigma^2$ varies with the range, *R*. That is, if the range of tones extends from *1* to *90* dB, the same two tones of *32* dB and *48* dB would generate bell curves with greater variances. Variance depends only on the range of values from which stimulus tones were drawn.

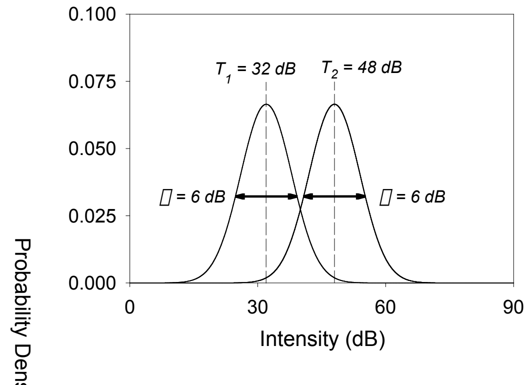


Figure 1. For stimuli $T_1$ = 32 dB and $T_2$ = 48 dB drawn from a given range of values (i.e. 1 to 90 dB), the curve describing subject errors will have the same shape for both.

When the number of tones presented to participants is symmetric about the value *R/2* (e.g. uniform or normally distributed), then the mean intensity of a tone in the range *1* to *R* dB is equal to *R/2*. Since variance, $\sigma^2$, varies with range, *R*, it varies also with the mean tone intensity, $I_{dB}$ (intensity measured in decibels). The manner in which $\sigma^2$ changes with $I_{dB}$ is the primary subject of this paper.

## CLASSICAL APPRAISAL OF LOUDNESS

Classically, loudness is measured subjectively, by presenting tones of various intensities to a subject who assigns numbers to each tone in accordance with his/her experience of loudness. Thus, a tone that sounds twice as loud will be assigned a number twice as great. The subjects' individual numerical scales are then normalized so that a tone of frequency 1000

Hz at 40 dB SPL will have a loudness value of one *sone*.

When data from many hundreds of participants are averaged, it has been found by very many investigators that the relationship between loudness, $L$, and intensity, $I$ (in watts.m$^{-2}$), is quite closely described (for $I_{dB} > 10$ dB) by a power function of the form

$$L = kI^n$$

where $k$ is a scaling factor and the exponent $n$ has been found to be relatively constant for tones of a given auditory frequency. That is, a full logarithmic plot of $L$ vs. $I$ will be linear with slope equal to $n$. For tones of *1000* Hz, $n$ takes the value of about *0.28*.

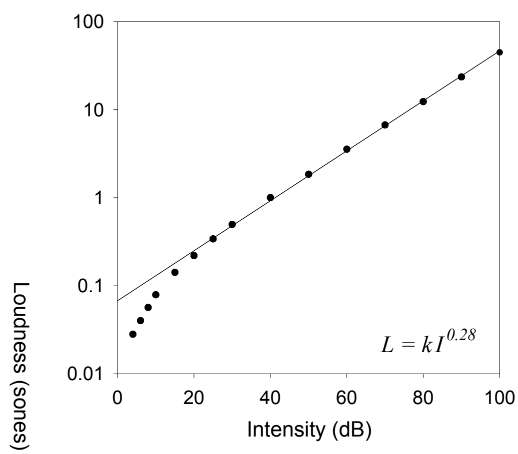


Figure 2. Loudness (in sones) vs. intensity (in dB) for stimulus tones at 1000 HZ. Data from Hellman and Zwislocki (1961).

We recall that $\sigma^2$ is the square of the error made by human participants in identifying the loudness of tones whose mean sound intensity is $I_{dB}$.

Suppose we now make a full logarithmic plot of $\sigma^2$ vs. $I$ . In Figure 3 we see such a graph for one subject drawn from a set of experiments carried out in our laboratory (Norwich et. al., 1998) with mean intensities *5, 15, 25, 35* and *45* dB. We note that a mean intensity of *15* dB corresponds to tones drawn from a range of 1-30 dB. The slope of the regression line is equal to *0.26*, and we can see that it follows the contour of the loudness graph in Figure 2. The value of the slope of the regression line is very close to the expected $n$-value of *0.28*.

Hence we see that there is experimental evidence supporting the hypothesis that the ordinates of the graphs in Figures 2 and 3 can be related by the equation

$$\sigma^2 \propto L.$$

That is, loudness, $L$, measured subjectively in the traditional manner, is proportional to $\sigma^2$, the objectively measured variance of the probability density governing errors in identification of tones.

The theoretical derivation of the above equation has been given by Norwich and Sagi (2002).

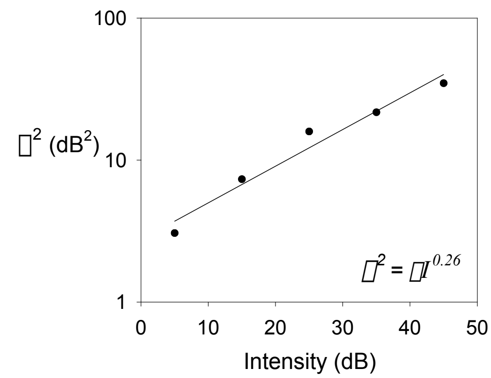


Figure 3. Subject response error (in dB$^2$) vs mean intensity (in dB).

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