ESTIMATION OF THE ORTHOTROPIC ELASTIC PROPERTIES OF THE RAT EARDRUM

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INTRODUCTION

Finite-element (FE) models of the eardrum have been developed to understand its contribution to the impedance-matching function of the middle ear [1]. Impedance matching permits the efficient transmission of sound from low-impedance air in the ear canal to high-impedance fluid in the cochlea. In principle, accurate FE models can also be used to study the effects of surgical interventions such as eardrum grafting. The accuracy of FE models of the eardrum is highly dependent on the elastic properties specified in the models. Refinements in techniques for measuring these properties would not only improve current models but would also permit quantification of changes in these parameters with age in order to correlate them with hearing performance. Several investigators have modeled the eardrum as a linearly elastic isotropic structure, and have measured the Young's modulus of the eardrum. However, the ultrastructure of the eardrum suggests that it is best modeled as a linearly elastic orthotropic material because the eardrum contains two orthogonal sets of fibres. No attempts have been reported in the literature to measure the eardrum's orthotropic elastic parameters *in situ*. In this paper we present a novel inverse FE technique to estimate these parameters. *In silico* experiments indicate that the method is robust.

METHOD

Indentation Technique

Indentation testing can be used to measure tissue elastic parameters. This technique involves indenting the tissue sample and acquiring the corresponding force-displacement data [2]. Using these data, the elastic properties can be estimated with an inverse FE technique that involves optimization of a FE model so that predicted force-displacement results match measured data. This technique has only been applied to measure the Young's modulus of the eardrum, which assumes that the eardrum is isotropic. In this work, to assess the performance of the indentation technique for measuring the eardrum's orthotropic parameters, indentation of the pars tensa of a rat eardrum specimen was simulated. In this *in silico* experiment the indenter was assumed to be perpendicular to the surface at the contact point. The experiment assumes using a spherical-ended indenter for indentation. The indentation was assumed to be conducted on the pars tensa while the malleus was immobilized to isolate the eardrum from the ossicular and cochlear loads [2], thus simplifying FE modeling of the eardrum as described below.

Pressurization Technique

A pressurization test can also be employed to measure the elastic properties of the eardrum using an inverse FE technique. This involves pressurizing the eardrum and acquiring 3D shape measurements. An FE model is then optimized so calculated pressurized shapes match measured ones. Similar to indentation, the malleus bone must be immobilized to isolate the eardrum from the effects of the ossicular and cochlear loads. Using the pressurization system shown in Figure 1, the pressure can be generated and applied manually to the eardrum. In this work a pressurization test was simulated *in silico.* This simulation assumes that the eardrum was uniformly pressurized from 0 kPa up to 4 kPa in increments of 0.5 kPa.

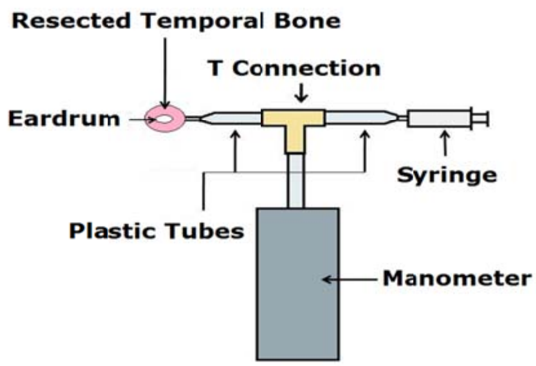

Figure 1: Schematic of pressurization system.

FE Model Construction

The first step to construct a FE model of the rat eardrum is measuring its 3D shape. To obtain shape data, Fourier transform profilometry (FTP) [3], a non-contact optical 3D measurement method, can be used. This involves projecting a grating pattern onto the specimen and acquiring the corresponding deformed grating pattern by a CCD camera. This pattern is then processed by a computer to determine the shape. In this *in silico* investigation, we a obtained a realistic eardrum shape using a commercial Fourier transform profilometer that has a depth-measuring accuracy of 10 µm (model MM-25D from Opton Company Limited, Seto, Aichi, Japan). FTP requires a diffusely reflecting surface with high contrast. Therefore, the eardrum was coated with a thin layer of white ink before measuring its shape because the eardrum is not diffusely reflecting.

The output of the Fourier transform profilometer was a 3D surface represented by a mesh and a high resolution digital image. The 3D surface mesh consists of a dense cloud of triangulated points representing the surface being measured. The high resolution digital image was used to manually define the boundary of the tympanic ring, pars tensa, pars flaccida, and the outline of the manubrium on the measured 3D surface.

A FE model of the eardrum was then generated from the cloud of points using the Trans-Finite Interpolation (TFI) method [4]. The TFI method involves a transformation of a FE mesh from a rectangular computational domain to an arbitrarily shaped physical domain. In order obtain a suitable FE mesh, the cloud of points representing the eardrum was broken down into fourteen zones, and a TFI-based technique was developed that transformed a unit square to each zone of the eardrum. Furthermore, elements of the pars tensa were aligned consistently with the orientation of the radial and circumferential collagen fibers, which is necessary for orthotropic model parameter reconstruction. Finally, a quadrilateral FE mesh with good quality was generated, which is shown in Figure 2. The Abaqus finite element software (Simulia Inc., RI, USA) was used to model the eardrum with four-noded S4 shell elements.

To model thin orthotropic materials such as the eardrum, four independent in-plane elastic constants are needed which are longitudinal (E_X) and transverse (E_Y) Young's moduli, in-plane shear modulus (G_{XY}), and Poisson's ratio. We focus on estimating E_X, E_Y, and G_{XY} for the pars tensa because of its role in transferring load from the eardrum to the bones. The thickness of the pars tensa was measured from micro-CT image and an average value of 12 µm was assigned to the model. The same thickness was taken for the pars flaccida. Furthermore, the pars flaccida was assumed to be isotropic and more compliant than the pars tensa. A Young's modulus value equal to one-twentieth of the expected longitudinal Young's modulus of the pars tensa was assigned to it. The ligament separating the pars tensa from the pars flaccida was modeled as a boundary between them. The ligament's thickness was assumed to be 12 µm and its Young's modulus was assumed to be the same as that of other ligaments [5]. The Young's modulus of the manubrium was assumed to be 15 GPa, which is the same as that of typical cortical bone [6]. The manubrial thickness was set to 100 µm based on a micro-CT scan. All tissues were assumed to have a Poisson's ratio equal to 0.3. The superior boundary of the manubrium and the tympanic ring were assumed to be fully clamped. However, the rest of the manubrium was tightly coupled to the eardrum and was free to move with it.

For the indentation technique, the eardrum was modeled as a deformable shell as described above while the indenter was modeled as a rigid surface.

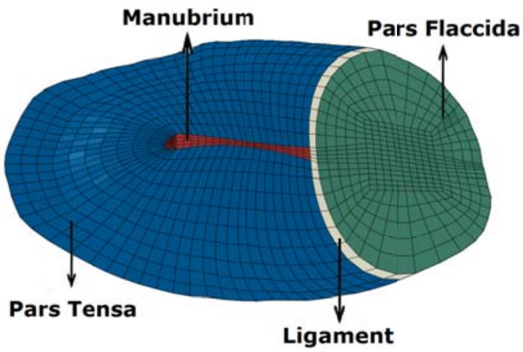

Figure 2: Sample FE model of the rat eardrum.

<u>Optimization</u>

A non-linear inverse problem formulated as an optimization problem can be solved in order to find the orthotropic elastic parameters using both techniques. Orthotropic values were found by optimizing two cost functions, one specifically designed for the indentation technique and the other for the pressurization technique. The cost function for the indentation technique is the sum of squared reaction force differences between simulated and experimental data leading to the following constrained optimization problem:

$$Min.\ C_{in1} = \sum_{i=1}^{n_1} (f_i^{ex} - f_i^{s})^2 \qquad (1)$$
$$Subject\ to \begin{cases} E_{XL} \le E_X \le E_{XU} \\ E_{YL} \le E_Y \le E_{YU} \\ G_{XYL} \le G_{XY} \le G_{XYU} \end{cases}$$

Here, f_i^{ex} and f_i^{s} denote the experimentally acquired and simulated reaction forces, respectively, for point i along the force-displacement curve. The number of points along the curve is specified by n_1. E_{XL} and E_{XU} are the lower bound and upper bound of the longitudinal Young's modulus, respectively; E_{YL} and E_{YU} are the lower bound and upper bound of the transverse Young's modulus; and G_{XYL} and G_{XYU} are the lower bound and upper bound of the in-plane shear modulus.

The cost function for the pressurization technique is the sum of squared nodal z-coordinate differences between simulated and experimental data leading to the following constrained optimization problem:

$$Min.\ C_{p2} = \sum_{i=1}^{n_2} (z_i^{ex} - z_i^{s})^2 \qquad (2)$$
$$Subject\ to \begin{cases} E_{XL} \le E_X \le E_{XU} \\ E_{YL} \le E_Y \le E_{YU} \\ G_{XYL} \le G_{XY} \le G_{XYU} \end{cases}$$

Here, z_i^{ex} and z_i^{s} are the experimentally acquired and simulated surface shape z-coordinates, respectively, for node i. The number of the points on the surface is specified by n_2.

Each cost function was minimized to find the optimal orthotropic elastic parameters using a variant of the Nelder-Mead simplex method [7]. The optimization process was terminated when the tolerance for the cost function values and orthotropic parameter values were small enough.

<u>*In silico* Method Validation</u>

Since the orthotropic elastic parameters of the rat eardrum are not known, the two methods were tested on synthetic computer-generated data simulating the indentation and pressurization experiments with various levels of Gaussian noise. These data were generated using an FE model that was created using the measured rat eardrum shape data. Arbitrary orthotropic parameter values referred to as "ground truth values" were assigned to the model.

Another model generated from the same shape data but with unspecified parameter values was then optimized using the above techniques.

RESULTS

The ground truth values were recovered with both techniques without any error in the absence of noise. Values of in-plane shear modulus recovered by the indentation and by the pressurization techniques are shown in Figure 3 and Figure 4, respectively, for various levels of Gaussian noise. Initialization values, optimization algorithm parameters, and FE models are the same for both figures. As expected, these figures show that the recovered values are closer to the ground truth values with smaller levels of noise. The pressurization technique has an accuracy in excess of 90% when the SNR (signal-to-noise ratio) is 2 or greater. For the indentation technique, an SNR greater than 200 is required to achieve over 90% accuracy. The above estimation method is valid over a wide range of initialization values used with the optimization algorithm from half the ground truth values to twice the ground truth values.

The behaviour of the remaining elastic orthotropic parameters (longitudinal and tangential Young's moduli) was found to be similar with respect to SNR.

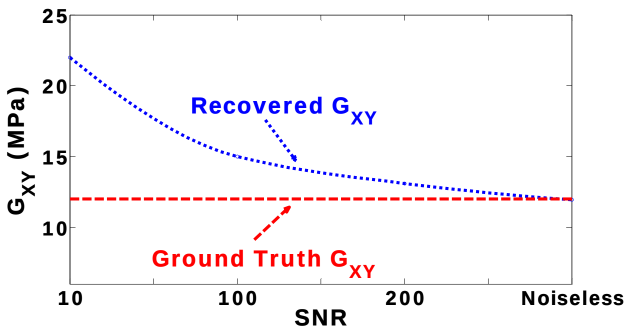

Figure 3: Ground truth and recovered value of in-plane shear modulus by indentation test with different levels of Gaussian noise.

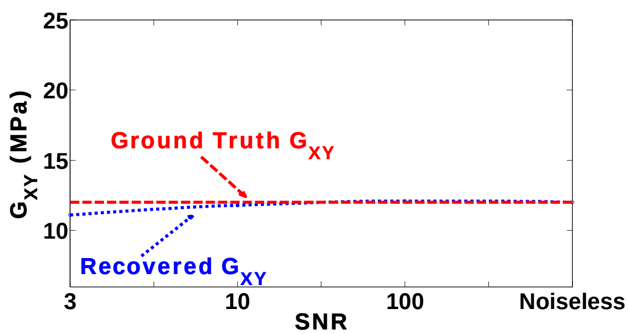

Figure 4: Ground truth and recovered value of in-plane shear modulus by pressurization test with different levels of Gaussian noise.

CONCLUSION

Two techniques were developed to estimate the orthotropic elastic parameters of the rat eardrum, an indentation technique and a pressurization technique. The pressurization technique is more robust to simulated noise than the indentation technique, and can achieve an accuracy in excess of 90% for an SNR of 2 or greater. The indentation technique requires an SNR of 200 or greater for the same level of accuracy. Although in general the simplex optimization method requires initialization values that are not very different from the correct moduli values, both the indentation and pressurization techniques as described here were robust to a wide range of initialization values. In conclusion, promising results were obtained and the developed techniques will be used with experimental data.

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