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NONLINEAR VISCO-ELASTIC MODELS APPLIED TO HUMAN MEDIAL COLLATERAL LIGAMENTS

Claudia Blandford[1]
[1]Department of Mechanical and Materials Engineering, Faculty of Engineering
the University of Western Ontario, London, ON

INTRODUCTION

Ligaments exhibit time and history dependent mechanical behaviours such as stress relaxation. The most commonly used model to describe these viscoelastic properties is Fung's quasilinear viscoelastic theory[1]. Although useful, this model does not account for the strain dependent rate behaviour of stress relaxation proven by Provenzano[2] (2001). This property has been better modelled in several studies with general nonlinear models[3,4,5,6,7]. Provenzano et. al (2002)[5] had particular success by applying the Schapery method[8] and the modified superposition method (MSM) (Findley 1976)[9] constitutive models to rat medial collateral ligaments (MCL) stress relaxation data. To the author's knowledge only one other study has attempted to model stress relaxation of a human MCL[4] and this study will attempt to apply Schapery method and MSM to human MCL data from Bonifasi Lista's 2005 study[10].

MATERIALS AND METHODS

Stress relaxation data of a longitudinally loaded human MCL was extrapolated from Bonifasi-Lista (2005) where the experimental methods are thoroughly described. In summary, the experiment consisted of incremental stress-relaxation tests where the specimen was stretched to first equilibrium strain level (1.6%) at 1%/s, allowed to stress relax, and then subjected to sinusoidal oscillations for a separate study. The protocol was then repeated for two other strain levels (2.4% and 3.2%). The stress relaxation data of this study is illustrated in figure 1.

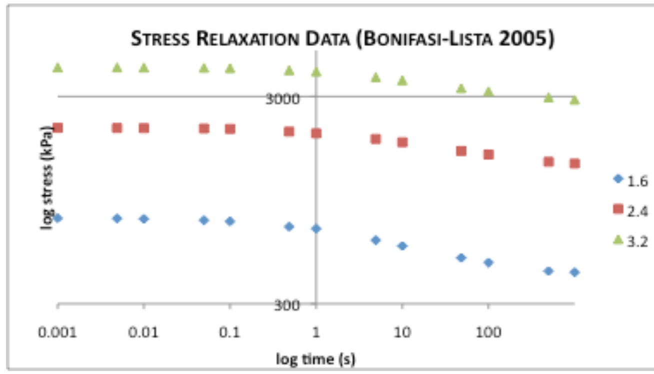

Figure 1: Stress relaxation data extracted from Bonifasi Lista (2005)[10].

This data was used to derive the parameters for the two following models:

Schapery Method

Schapery's nonlinear viscoelastic theory is described in detail in Schapery 1969[8] and is derived from irreversible thermodynamic principles. In a one-dimensional case (uni-axial loading) the integral expression is:

$$\sigma(\varepsilon,t) = h_e(\varepsilon)E_e\varepsilon + h_1(\varepsilon)\int_0^t \Delta E(\rho(t)-\rho'(\tau))\frac{dh_2(\varepsilon)\varepsilon}{d\tau}d\tau \quad (eq1)$$

with ρ, the reduced time, defined as:

$$\rho = \int_0^t \frac{dt'}{a_e[\varepsilon(t')]} \quad (eq2)$$

where h_e, h_1, h_2, and a_e are strain dependent material properties that vary due to strain effects in the Helmholtz free energy equation. For this application a_e and h_1 are set to unity[8]. E_e is the final elastic modulus in the experimental time frame and ΔE is the transient modulus that can be modelled as a power law:

$$\Delta E(\rho) = C\rho^{-n} \quad eq(3)$$

Substituting a Heaviside function of ε_0 into eq1 as the strain history results in:

$$\sigma(\varepsilon,t) = h_e(\varepsilon)E_e\varepsilon_0 + h_2 C\varepsilon_0 t^n \quad eq(4)$$

Where E_e and ε can be found with experimental data and h_e and h_1 can be found either with experimental data or by curve fitting.

Modified Superposition Method (MSM):

MSM best described in Findley (1976)[9] takes the form of a non-separable strain-dependent power law:

$$E(\varepsilon,t) = A(\varepsilon)\varepsilon_0 t^{B(\varepsilon)} \quad eq(5)$$

where $A(\varepsilon)$ represents the initial elastic modulus as a function of strain found with experimental stress-strain data and $B(\varepsilon)$ is the strain-dependent rate of stress relaxation and can be determined by curve fitting stress relaxation rate-strain data.

PARAMETER ESTIMATION AND RESULTS

Schapery Method

E_e was found to be 1540.8 kPa by calculating the tangent modulus of the stress-strain data at t=1000s. C and n were determined by two ways. In Provenzano 2002[5], the authors determined C and n by setting h_e and h_2 to unity and curve fitting the 1.6% strain stress relaxation data with eq(4). This resulted in a C = -1138.3 and an n = 0.001181. Although this makes mathematical sense, these values should represent the transient modulus (recall eq(3)) which is more accurately modelled with a decaying power law. Thus, the transient modulus was found by calculating the tangent modulus of stress-strain data at each time point, and by fitting this modulus-time data with a power curve to obtain C = 1919.7 and n = -0.024. By then setting C and n to these values and fitting eq(4) to the stress relaxation data we can find the values of h_e and h_2 of each strain. The functions $h_e(\varepsilon)$ and $h_2(\varepsilon)$ were found by then fitting the h-strain curve with a logarithmic and exponential curve respectively (curves of highest R^2 value) as illustrated in Figure 2.

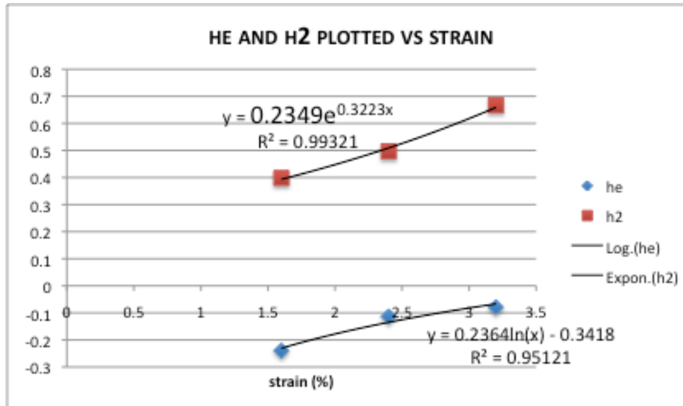

Figure 2: h_e and h_2 as a function of strain.

Adding the final and tangential stresses completes the model as follows

$$\sigma(\varepsilon,t) = [0.2364\ln(\varepsilon) - 0.342]E_e\varepsilon + [0.2349e^{0.3223\varepsilon}]1919.7t^{-0.024} \quad eq(6)$$

The model is plotted with experimental data in Figure 3. The R^2 values for the 1.6, 2.4 and 3.2% strain cases were 0.911, 0.862 and 0.859 respectively.

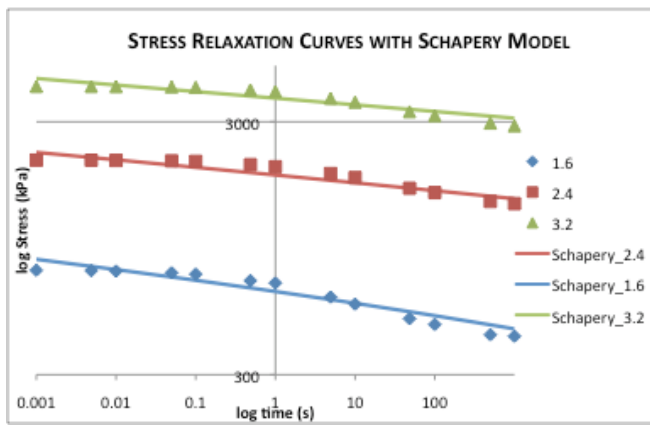

Figure 3: Experimental data with Schapery method model.

Modified Superposition Method

$B(\varepsilon)$ was found by fitting a polynomial curve to the stress relaxation rates – strain data. The stress relaxation rate is defined as the slope on the log-log stress-time graph and was found to be -0.0496, -0.0312 and -0.0281 for 1.6, 2.4 and 3.2 % strain respectively. This resulted in:

$$B(\varepsilon) = -0.012\varepsilon^2 + 0.0709\varepsilon - 0.1325 \quad eq(7)$$

A(ε) was found by calculating the initial young's modulus (t=0.001s) at each strain and by fitting a linear curve (best fit at R^2 = 0.99997). The initial young's moduli for 1.6, 2.4 and 3.2 % strain were 389.38kPa, 776.16kPa and 1155.6kPa respectively. A(ε) was thus found to be:

$$A(\varepsilon) = 478.89\varepsilon - 375.62 \quad eq(8)$$

Thus:

$$\sigma(t,\varepsilon) = [478.9\varepsilon - 375.62]\varepsilon \times t^{-0.012\varepsilon^2 + 0.0709\varepsilon - 0.1325} \quad eq(9)$$

The model is plotted with experimental data in Figure 4. The R^2 values are 0.881, 0.855 and 0.849 for 1.6, 2.4 and 3.2 % strain respectively.

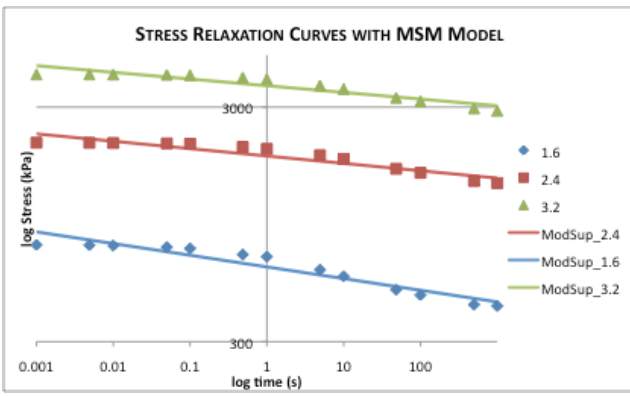

Figure 4: Experimental data with MSM model.

DISCUSSION AND CONCLUSION

Both models were able to successfully describe ligament stress relaxation, which is useful in making predictions on intermediate ligament behaviour and implementation in finite element analysis. Information on ligament parameters is also evident by these models. The Schapery model's C and n indicate how the elastic modulus changes over time Furthermore, the fact that $h_2(\varepsilon)$ approaches zero suggests that relaxation rate reduces with increased strain with diminishing returns. MSM illustrates rate dependence on strain through B(ε) and clearly highlights the initial moduli. Overall, MSM is advantageous in that the stress-relaxation behaviour is modelled with a single term, whereas the Schapery model fits the experimental data more closely as shown by the lower R^2 value.

REFERENCES

[1] Fung. "On mathematical models of stress-strain relationship for living soft tissue". *Mech Compos Mater.*, 11:726-40. 1975

[2] P. Provenzano, R. Lakes, T. Keenan, R. Vanderby Jr. "Nonlinear ligament viscoelasticity". *Ann Biomed Eng*, 29:908-14. 2001.

[3] R. Hingorani, P. Provenzano, R. Lakes, A. Escarcega, R. Vanderby Jr. "Non-linear viscoelasticity in rabbit medial collateral ligament." *Ann Biomed Eng*, 32:306-12. 2004

[4] E. Pena, J. Pena, M. Doblare. "On modeling nonlinear viscoelastic effects in ligaments". *J Biomech*, 41:2659-66. 2008

[5] P. Provenzano, R. Lakes, D. Corr, R. Vanderby. "Application of nonlinear viscoelastic models to describe ligament behavior". *Biomech Model Mechan*, 1:45-7. 2002

[6] R. Sopakayang, R De Vita. "A mathematical model for creep, relaxation, and strain stiffening in parallel fibered collagenous tissues". *Med Eng Phys,* 33:1056-63. 2011

[7] P. Vena, D. Gastaldi, R. Contro. "A constituent-based model for the nonlinear viscoelastic behavior of ligaments". *J Biomech Eng-T ASME*, 128:449-57. 2006

[8] R. Schapery. "On the characterization of nonlinear viscoelastic materials". *Polym Eng Sci*, 9:295-310. 1969

[9] W. Findley, J Lai, K. Onaran. "Creep and relaxation of nonlinear viscoelastic materials". Dover, NewYork. 1976

[10] C. Bonifasi-Lista, S Lakez, M. Small. "Viscoelastic properties of the human medial contralateral ligament under longitudinal, transverse and shear loading". *Orthop Res*, 23:67-76. 2005