NUMERICAL MODELING OF RED BLOOD CELLS SEDIMENTATION USING POPULATION BALANCE MODELING

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ABSTRACT

This work describes the development of a two-dimensional CFD model for Red Blood Cells (RBC) sedimentation. Population balance dynamic equation is solved for RBCs including RBCs aggregation. Rate of RBCs aggregation is obtained from literature and time of sedimentation is compared to what is on literature. Using this model cell free zone formation and RBC's network formation could be modeled and be seen. Link between aggregation rate and sedimentation time is an important matter which is been described in this paper.

INTRODUCTION

The use of erythrocyte sedimentation rate (ESR) in clinical investigations goes back to the works of Fahraeus [1] and Westergren [2] who have proven the diagnostic potential of this phenomenon. Nowadays, this inexpensive test still remains a way to diagnose conditions associated with multiple myeloma, acute and chronic inflammation, malignant, autoimmune and sickle-cell diseases [3-5]. These diseases may cause anemia and are often associated with an enhanced level of normal and abnormal plasma proteins, which promote red blood cell (RBC) aggregation [5].

In this paper population balance Equation (PBE) is used to model RBC aggregation. This model was previously been used for bubbly flows were breakup and coalescence of bubbles are important [6]. Current modelings of RBC aggregation just consider finite number of RBCs and it also not considers the blood flow and its effects on aggregation. Generally PBE could give a macroscopic view of aggregation which other models were unable to produce.

Aggregation rate has a direct relation with sedimentation time [1], this new model is developed to find this relation. This is the first step to develop a complete numerical code for blood flow as a two phase flow where both non-Newtonian and Newtonian characteristic of blood could be simulated. Current results for simple sedimentation geometry and experiment show promising outcomes from this model.

POPULATION BALANCE EQUATIONS (PBE)

In general, the PBE is a balance equation of the number density of some particle property. In this application, the particle property is RBCs aggregated size (d). The PBE in its most general form is given by Ramkirishna [7] as:

$$\frac{\partial n(d,x,t)}{\partial t} + \nabla_d \cdot \dot{D} n(d,x,t) + \nabla_x \cdot u_p n(d,x,t) = B(d,x,Y,t) - D(d,x,Y,t) \quad (1)$$

In this equation $n(d,x,t)$ is the density number of particles of diameter d and is function of the particle location x and time t. $\dot{D}$ is the growth rate of diameter without any breaking or collision and u_p is the particle velocity. In the right hand side of the equation, B stands for the birth and D for the death rate of particles of size d. The dependence on parameter Y is introduced to describe the birth and death rates of particles influenced by the continuous phase.

The second term in Eq. 1, which is the effect of mass transfer on particle growth, is negligible. If we discretize this equation for particle size dimension, Eq. 1 becomes:

$$\frac{\partial n_i}{\partial t} + \nabla u_i n_i = B_i - D_i \quad (2)$$

This approach is called method of classes in which particles are classified according to the diameter. Each class i is consisted of particle with volume of v_i and diameter of d_i. In the present work the classes are made in a way that v_{i+1} is twice larger than v_i. For our application, diameters start with one single RBC, second size is two RBCs together, and third size is four RBCs together and so on.

In Eq. 2, u_i is the velocity of a particle with a diameter of d_i. The velocity is a function of particle diameter, domain properties and continuous fluid properties which will be describe in more details in the following section.

In Eq. 2, $B_i - D_i$ is the source term for class i which is affected by the break up and aggregation of RBC's cluster in other classes. This term could be stated as:

$$S_i = B_i - D_i = \left[B_B - D_B + B_C - D_c\right]_i \tag{3}$$

In which, for class i, B_B indicates the birth of particle in class i due to breakage of larger particles, D_B the death of particles caused by breakage, B_C the birth of particle resulted by smaller particle aggregation and D_C the particle death as they aggregation with others. Waghmare [8] suggested the following equations to describe these terms:

$$\begin{aligned} B_B(i) = \sum_{k=i+1, i\neq N}^{N} \Omega_B(k,i) + \sum_{K=1, i\neq N}^{i} yb_{i+1,k}\Omega_B(i+1,k) \\ + \sum_{k=} \left(1 - yb_{i,k}\right)\Omega_B(i,k) \quad for\ i = 1,....N \end{aligned} \tag{4}$$

In which when $k > i$, $yb_{i,k} = 2^{1+k-i}$ and

$$D_B(i) = \sum_{k=1}^{i-1} \Omega_B(i,k) \quad for\ i = 2,....N \tag{5}$$

$$\begin{aligned} B_c(i) = \sum_{j=1, j\neq N}^{i-1} yc_{i,j}\Omega_c(i, j) \\ + \sum_{j=1}^{i-1} (1 - yc_{i-1,j})\Omega_c(i-1, j) \quad for\ i = 2,....N \end{aligned} \tag{6}$$

In which when $i > j$, $yc_{i,j} = 1 - 2^{j-i}$ and

$$D_c(i) = \sum_{j=1}^{N-1} \Omega_c(i, j) + \Omega_c(i,i), \quad i = 1,..., N-1 \tag{7}$$

Term $\Omega_B(i,k)$ calculates the breakage rate of particles in class i to produce particle in class k while $\Omega_c(i, j)$ calculates the aggregation rate of particle due to collision of particles in class i and j. The first term in the second side of the Eq. 4 indicates the birth rate of particle in class i due to breakage of larger particle in class k ($k > i$). As a result this term should be calculated for $k = i + 1$ to k=N. Same logic applies to Eq. 5 to Eq. 7.

An important assumption in this model is that the smallest particle ($i = 1$) can't break (single RBC) into any smaller particle and also the largest ones ($i = N$) wouldn't make any larger particle as they collide to other particles.

In this paper, as we do not have any breakage in domain we could neglect birth and death due to breakage. So the important parameter that should be calculated from experimental results is $\Omega_c(i, j)$, which it is rate of aggregation of RBCs. Calculating this parameter is described on experimental result section.

CONTINUOUS FLUID GOVERNING EQUATION

Continuity and momentum in *x* and *y* directions are as follow:

$$\frac{\partial(\rho\alpha u)}{\partial x} + \frac{\partial(\rho\alpha v)}{\partial y} = 0 \tag{8}$$

$$\begin{aligned} \frac{\partial(\rho\alpha uu)}{\partial x} + \frac{\partial(\rho\alpha uv)}{\partial y} = \\ -\frac{\partial p}{\partial x} + \frac{\partial}{\partial x}\left(\mu\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) + F_{gl} \end{aligned} \tag{9}$$

$$\begin{aligned} \frac{\partial(\rho\alpha uv)}{\partial x} + \frac{\partial(\rho\alpha vv)}{\partial y} = \\ -\frac{\partial p}{\partial y} + \frac{\partial}{\partial x}\left(\mu\frac{\partial v}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu\frac{\partial v}{\partial y}\right) - \rho g + F_{gl} \end{aligned} \tag{10}$$

In these equations α is volume fraction of liquid, F_{gl} is the source term for reaction force of particles applied to liquid [9].

RBC velocity, which is required for solving the PBE, is calculated by balancing the gravity and drag forces acting on the RBC. That is

$$f_D = C_D \frac{\pi d_B^2}{4} \frac{1}{2} \rho_L (u_G - u_L) |u_G - u_L| = v_B(\rho_G - \rho_L) g \qquad (11)$$

Drag coefficient for RBC is variable. For this study we assumed that the RBC has an elliptical shape and by multiplying the spherical drag coefficient to shape factor we could have a constant drag coefficient, the elliptical shape factor is 0.6 [10].

NUMERICAL MODELING

Boundary conditions, domain geometry and applied grid are shown in Fig-1.

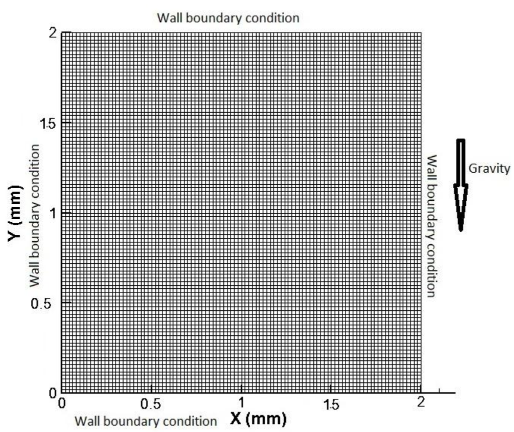

Figure 1- Boundary conditions and Domain grid

For solving continuous phase mass and momentum consistency, a CFD numerical FORTRAN code is developed. SIMPLEC method is used for discrediting these equations and the code is checked with standard benchmarks like cavity.

Boundary condition for CFD code is no slip and no penetration and for PBE is no penetration. Also for each mesh there is a saturation condition. When this condition is activated no particle may enter that node. Normally this condition will not be activated in flow. But in this model where there is sedimentation and RBCs pile up at the end of the domain, this condition is necessary.

PBE numerical code is developed separately and coupled with the CFD code. This coupling is shown in Fig-2.

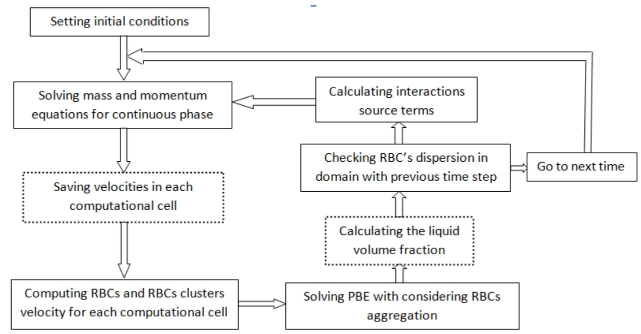

Figure-2- CFD and PBE numerical coupling

For initial condition, the volume fraction of single RBCs is given as 0.3(Hematocrit=30%)[5]. Density of RBCs is set to be 1120 and plasma density (continuous phase) is set to be 1001. Twelve classes are set for PBE modeling and gravity is the only reason for RBCs movement. And 0.4 was chosen as a rate of sedimentation[5].

For grid independency 3 grids were used, 200x200, 500x500 and 1000x1000. For studying quantitatively, velocity of the layer between upper part of the domain and lower part were compared (No RBC zone growth). Comparing this velocity showed that 500x500 has a same result as 1000x1000 with less than 1% error.

RESULT AND DISCUSSION

Numerical results are shown in figure 3 for different times. Numerical results show formation of a structural network of RBCs and cell free zones between RBC's clusters. Theses features are in agreement with experimental observations [5,11,12]. These RBCs structures change the balance between drag force and gravity. Higher mass cause higher gravity force, clustering cause higher drag coefficient. Comparing Non aggregating sedimentation with current experiment shows that we have faster sedimentation considering aggregation.

Wall effect is another phenomenon that should be fully studied. As the simulation shows RBC clusters near wall have difficulty to sediment. This might come from no slip condition and slower plasma flow in that part. Although we don't have much of the plasma flow (continuous phase flow) but that small movement of plasma is the reason for not having a complete structure of RBCs clusters in region between wall and center line.

CONCLUSION

In this paper numerical simulation of RBCs sedimentation was studied. Gravity acts as an external force for RBCs and is the only source of motion. This model is able to show a full formation of RBCs structures and sedimentation using PBE.

PBE could present a mesoscopic model that can links micro behavior (aggregation) to macro blood properties (sedimentation). This model shows promising result for studying further more in this field such as microcirculation blood flow where RBC aggregation has a major effect on the flow.

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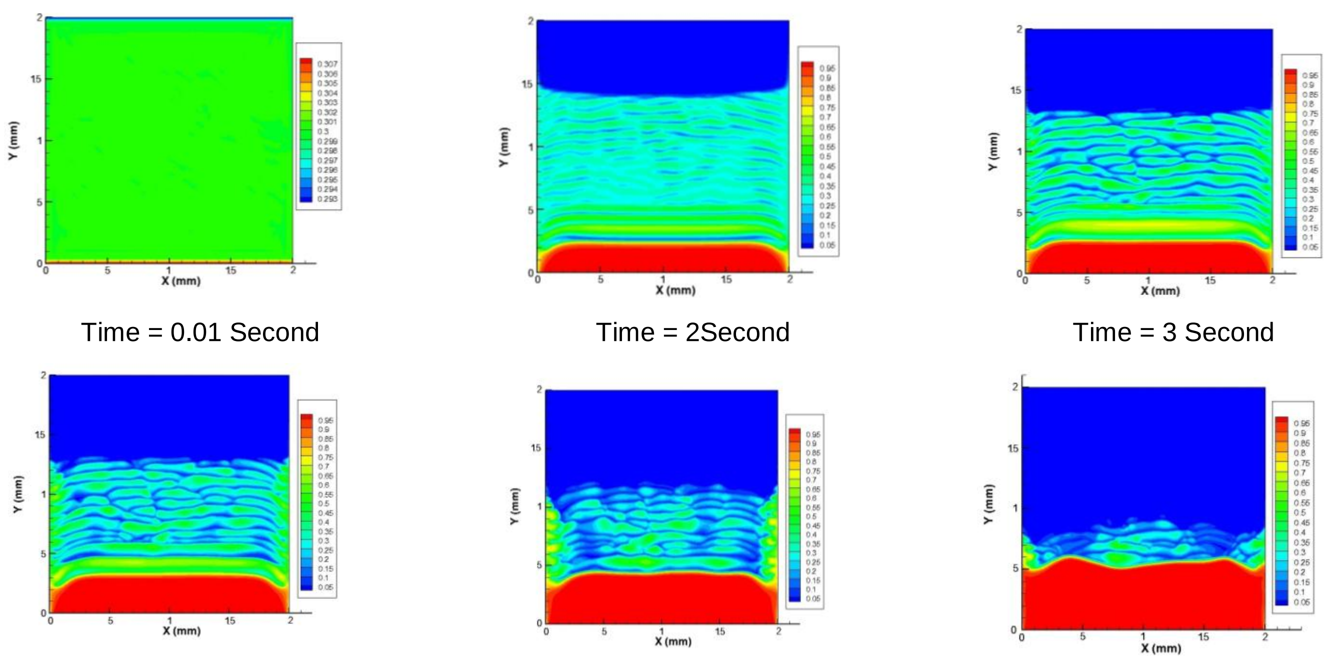

Time = 0.01 Second

Time = 2Second

Time = 3 Second

Time = 4 Second

Time = 5 Second

Time = 6 Second

Figure 3- Volume fraction of RBC in different times