ESTIMATION OF CONDUCTION VELOCITY DISTRIBUTION OF NERVE FIBRES BASED ON INVERSE PROBLEM FORMALISM IN ELECTRONEUROGRAPHY

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ABSTRACT- In the literature, it has been shown that the severity of Carpal Tunnel Syndrome (CTS) and other peripheral neuropathies depend on the condition of both slow and fast conducting fibres present in a nerve. In this context, it has become important to estimate the conduction velocity distribution (CVD) of both slow and fast fibres. In the proposed paper, compound nerve action potentials (CNAPs) are simulated and two different approaches that are based on the inverse problem formalism are used for estimating the CVD of nerve fibres. The first approach is concerned with the estimation of CVD of fast conducting fibres and the second approach is concerned with the estimation of CVD of slow conducting fibres. The performance of the CVD estimator is measured at different signal to noise ratios (SNRs) when random noises are added to the CNAPs.

Keywords – carpal tunnel syndrome, surface single fibre action potential, compound nerve action potential, conduction velocity distribution, delay distribution, conduction velocity distribution estimator

I. INTRODUCTION

In the literature, it has been shown that the severity of Carpal Tunnel syndrome (CTS) advances from large myelinated nerve fibres to small nerve fibres [1]. It is also shown in literature that A-delta fibres have a conduction velocity (CV) in the range 2 – 30 m/s and these are responsible for the sensation of cold and pain [1]. It is also shown in literature that an average sensory CV of 45 m/s or less suggests the presence of CTS [1]. As the severity of CTS and other peripheral neuropathies advances, a significant number of nerve fibres are slowed due to demyelination effect. Because of this effect, it is very important to know the conduction velocity distribution (CVD) of both slow and fast fibres.

In this paper, a technique is presented which estimates the delay distribution as well as the CVD of both slow and fast conducting fibres within a nerve. A bimodal distribution of conduction velocities is assumed for simulating compound nerve action potentials (CNAPs). After the CNAPs are simulated, two different mathematical approaches that are based on inverse problem formalism are used to build an estimator which estimates the CVD of nerve fibres [2]. The first approach is concerned with the estimation of delay distribution of fast conducting fibres whose conduction velocities lie in the range of 35 – 80 m/s. These fibres contribute to the main complex of the CNAP wave-shape. The second approach is concerned with the estimation of the delay distribution of slow conducting fibres whose conduction velocities lie in the range of 5 – 30 m/s. These fibres contribute to the late components of the CNAP wave-shape. The overall CVD of all the active fibres is obtained from the estimated delay distributions. A mean square error (MSE) is used for evaluating the performance of the estimator when different levels of noise are added to the simulated CNAPs.

II. THEORETICAL BACKGROUND

The estimation of CVD of nerve fibres is obtained in two parts. The first part deals with the estimation of CVD of fast conducting fibres that are contributing to the main complex of CNAP and it uses a least square optimization technique for estimating the CVD of fast fibres. The second part deals with the estimation of CVD of slow conducting fibres that contribute to the small late components of the CNAP. It uses a single realization of a non-stationary Poisson process. Here, the CNAP is defined to be a filtered Poisson process where the time-varying filter represents the Surface Single Fibre Action Potential (SSFAP) wave-shapes [2]. In both parts of the estimation procedure, the description of the wave-shape of the SSFAP is derived from a mathematical formulation given in [3] and is time scaled as described in [4].

The theoretical model for CNAP is obtained as a linear summation of Surface Single Fibre Action Potentials (SSFAPs) [4]. Hence, the CNAP is given by

$$CNAP(t,d) = \sum_{i=1}^{N} SSFAP_i\left(t-\tau_i; v_i\right) \tag{1}$$

where $CNAP(t,d)$ represents CNAP as a function of time, $SSFAP_i\left(t-\tau_i; v_i\right)$ represents SSFAP associated with a fibre with propagation velocity v_i ,

τ_i represents delay associated with propagation velocity v_i and propagation distance d, and N represents the total number of active fibres.

The inverse mathematical formalism for estimating delay distribution of nerve fibres is based on following equation:

$$CNAP(t,d) = \sum_{i=1}^{N} a(\tau_i;d) SSFAP_i(t-\tau_i;v_i \quad (2)$$

where $a(\tau_i;d)$ represents the delay distribution associated with propagation distance d. The delay distribution can be obtained using the following equation:

$$DELAY\_DIST = SSFAP^{-1} CNAP \quad (3)$$

where $DELAY\_DIST$ is a vector representing elements of delay distribution, $SSFAP$ is a matrix representing elements of Surface Single Fibre Action Potentials, and $CNAP$ is a vector representing elements of Compound Nerve Action Potentials.

III. METHODOLOGY

Simulation

A bimodal distribution of conduction velocities is assumed for simulating CNAPs [5]. The CVD assumed resembles a pathological condition where fast conductive fibres are severely damaged. A total of 3000 fibres have been selected for generating the bimodal distribution. About 60% of the total number of fibres is assigned to slow conducting fibres and 40% of the total number of fibres is assigned to fast conducting fibres. The first normal distribution has a mean of 15 m/s with a standard deviation of 3 m/s and the second normal distribution has a mean of 55 m/s with a standard deviation of 6 m/s (see Figure 1).

Ten CNAPs, each having duration of 50 msec, are generated from the above bimodal distribution. The SSFAPs are generated by convolving a second order derivative of signal source with a tissue filter impulse response. The signal source is obtained from the mathematical formulation given in [3] and is time scaled as described in [4]. The tissue filter impulse response used is described in [4].

Algorithm Implementation

The steps that are involved in the algorithm for developing the estimator for estimating the CVD of nerve fibres are as follows:

1. A CVD of bimodal nature is assumed for simulating CNAPs. SSFAPs are obtained as a result of convolution of second order derivative of temporal representation of source with a tissue filter impulse response.
2. The delay distribution of fast conducting fibres is obtained by minimizing a function $\|\mathbf{P}\mathbf{a}_1 - \mathbf{y}_1\|^2$ subject to the non-negativity constraint given by $a_1(\tau_n;d) \geq 0$. Here, $\mathbf{P}$ is a 1000×30 matrix representing SSFAPs with elements given by $P_{mn} = SSFAP_1(t_m - \tau_n; v_n)$, $\mathbf{a}_1$ is a 30×1 vector representing delay distribution with elements given by $a_{1_n} = a_1(\tau_n;d)$, and $\mathbf{y}_1$ is a 1000×1 vector representing CNAP with elements given by $y_{1_m} = \sigma^2_{y_1}(t_m;d)$. The above minimization problem is solved by using a non-negative least squares estimation subroutine given in MATLAB 7.0.1.
3. The delay distribution of slow conducting fibres is obtained by minimizing a function $\|\mathbf{Q}\mathbf{a}_2 - \mathbf{y}_2\|^2$ subject to the non-negativity constraint given by $a_2(\tau_n;d) \geq 0$. Here, $\mathbf{Q}$ is a 1000×30 matrix representing squared SSFAPs with elements given by $Q_{mn} = SSFAP_2(t_m - \tau_n; v_n)$, $\mathbf{a}_2$ is a 30×1 vector representing the intensity of the Poisson process realization of the delay distribution of slow conducting fibres with elements given by $a_{2_j} = a_2(\tau_j)$, and $\mathbf{y}_2$ is a 1000×1 vector representing variance of non-stationary Poisson process realizations of CNAPs with elements given by $y_{2_m} = y_2(t_m;d)$. The above minimization problem is solved by using a non-negative least squares estimation subroutine given in MATLAB 7.0.1.
4. The CVD is obtained from the above delay distributions using the equation $f_v(v) = d\,\frac{f_a(t)}{v^2}$ where $f_v(v)$ is the velocity probability distribution function,

$f_a(t)$ is the delay distribution, d is the propagation distance, and v is the corresponding velocity of propagation as described in [4]

5. Before applying step 3 and 4, a selection of conduction velocity range is made. For slow conducting fibres, a range is selected between 5 and 35 m/s, and for fast conducting fibres, a range is selected between 30 and 80 m/s.

IV. RESULTS AND DISCUSSION

Simulation Results

The following three plots display the assumed CVD of nerve fibres, the SSFAP at different velocities and the averaged CNAP simulated from the CVD.

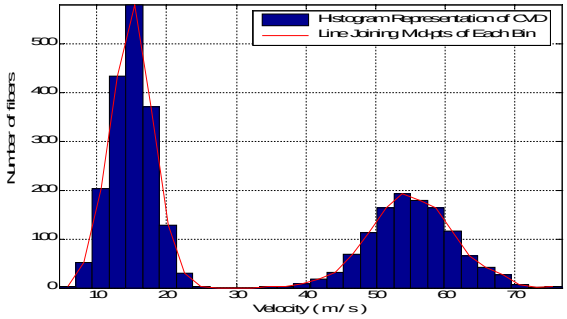

Figure1. Assumed CVD

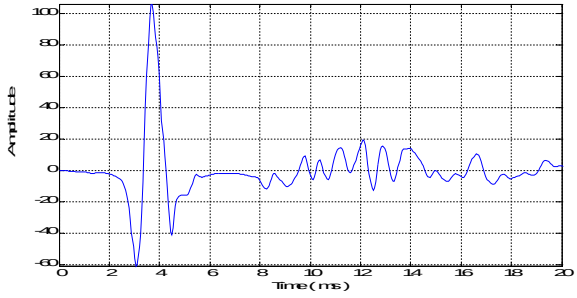

Figure2. Simulated Avg. CNAP at $SNR = \infty$

Figure 1 displays a histogram representation of bimodal distribution of nerve conduction velocities. X-axis represents the conduction velocities of nerves and Y-axis represents the number of active fibres. The mean and the standard deviation of Gaussian distribution of slow conducting nerve fibres are 15 m/s and 3 m/s respectively whereas the mean and standard deviation of Gaussian distribution of fast conducting nerve fibres are 55 m/s and 6 m/s respectively. Figure 2 displays an averaged CNAP with X-axis representing the time in millisecond and Y-axis representing the amplitude.

The solid line and the broken line in Figure 3 display the assumed CVD and the estimated CVD of all the conducting nerve fibres respectively at $SNR = \infty$. Figure 4 displays simulated CNAP in presence of Gaussian noise at $SNR = 50$. X-axis represents time in millisecond and Y-axis represents amplitude of the CNAP in presence of noise. The solid line and the broken line in Figure 5 display the assumed CVD and the estimated CVD of all the nerve fibres respectively at $SNR = 50$.

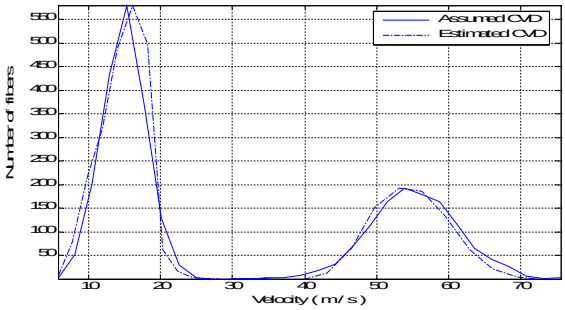

Figure3. Assumed CVD Vs Est. CVD at SNR = ∞

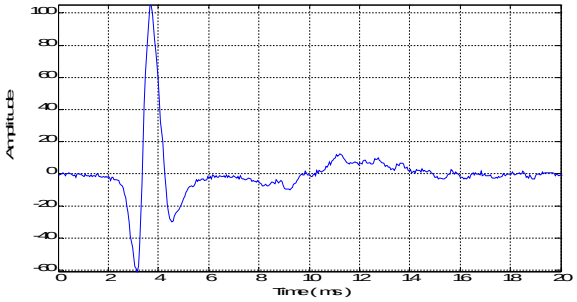

Figure4. Simulated Avg. CNAP at $SNR = 50$

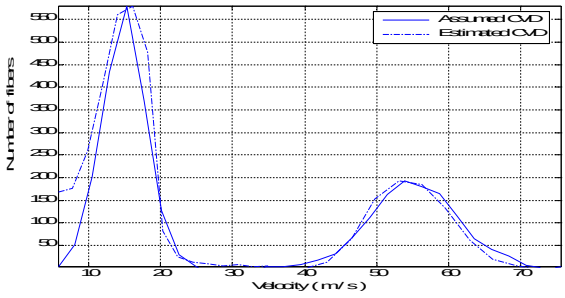

Figure5. Assumed CVD Vs Est. CVD at SNR = 50

Performance Measure

Simulations are run to evaluate the performance of the CVD estimator when random noise is added to the CNAP. The SNR is obtained by dividing the peak value of the averaged CNAP by the standard deviation of noise.

The performance of the CVD estimator is measured in terms of mean square error (MSE). The MSE is calculated between the assumed CVD and the estimated CVD of all the nerve fibres using the following equation:

$$MSE = \frac{\sum \left(Est\_CVD - Assumed\_CVD \right)^2}{\sum \left(Estimated\_CVD \right)^2} \quad (4)$$

Table 1: SNR Vs Mean Square Error

	SNR	***MSE***
1.	Infinity	0.3088
2.	50	0.3247
3.	20	0.6781
4.	10	0.7774
5.	5	0.8954

Table 1 displays different SNRs and corresponding mean square errors (MSEs) between assumed CVD and estimated CVDs at different levels of noise. It can be seen from Table 1 that as SNR is decreased from infinity to 5, the corresponding MSE values increase. It is also observed that when the SNR is decreased below 5, the CVD estimator had a difficulty in estimating the CVD of slow conducting nerve fibres.

V. CONCLUSIONS

It is concluded that the CVD estimator developed in this paper can estimate CVD of both slow and fast conducting fibres when a CNAP is available as input to the estimator. Since the mean square errors (MSEs) between the assumed CVD and the estimated conduction velocity distributions (CVDs) at different signal to noise ratios (SNRs) are very less, it can be concluded that the CVD estimator developed in this paper can reliably estimate CVD of very slow conducting fibres (5 – 30 m/s). The estimator developed in this paper can be used to estimate CVD of nerve fibres whose conduction velocities lie in the range 0.5 – 100 m/s.

The estimated CVD may be used by clinicians to obtain information about the condition and characteristic of nerve fibres and the information so obtained may be further used by clinicians in order to diagnose different stages of peripheral neuropathies.

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