

THE RELATIONSHIP BETWEEN THE POROELASTIC WAVE AND THE ELECTROKINETIC EFFECT FOR CANCELLOUS BONE

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INTRODUCTION

Poroelasticity has been employed for elucidating the bone fluid flow stimulating bone cells. So called "mechanobiology" explains how bone cells respond to mechanical stimuli and these bone cells communicate each other [1].

Bone remodeling process is a phenomenon how bone matrix is removed and formed by new bone, but the mechanism of bone remodeling process is unclear. Primary candidates for bone remodeling process are bone fluid flow, strain energy density, piezoelectricity, electrokinetic effects (or streaming potential), and so on. However it is not easy to detect these primary candidates experimentally, thus, in this paper, we suggest the charge density in electrokinetic effects (or streaming potential) can affect the fast and slow wave speed in cancellous bone by reformulating the general bone poroelasticity equation and the relationship between the fast and slow waves speed and the charge density is obtained.

METHOD

The governing equations for the poroelasticity theory are given by [2]

$$\rho_{11}\ddot{u}_i + \rho_{12}\ddot{U}_i - Nu_{i,jj} - (A+N)u_{j,ji} - QU_{j,ji} + b(\dot{u}_i - \dot{U}_i) + q_s\Phi_{,i} = 0 \quad (1)$$

$$\rho_{22}\ddot{U}_i + \rho_{12}\ddot{u}_i - (RU_{j,j} + Qu_{j,j})_{,i} - b(\dot{u}_i - \dot{U}_i) + q_s\Phi_{,i} = 0 \quad (2)$$

$$\{q_f(RU_{j,j} + Qu_{j,j}) - \varepsilon b[\dot{\Phi} + (\beta G / \varepsilon)]\Phi\}_{,ii} = 0 \quad (3)$$

,where N , R , Q , and A are poroelastic parameters, u and U are displacements of fluid and solid constituents, and the variable b is defined by $\phi^2 \mu / K$, where μ is the fluid viscosity, K is the permeability of bone fluid, ε is the permittivity, and the ϕ is the porosity. q_s is the charge density and Φ is the electric potential. After we define $P \equiv A + 2N$, the poroelastic parameters or, so called Biot parameters P , Q , and R are given by

$$P = \frac{\phi(K_s / K_f - 1)K_b + \phi^2 K_s + (1 - 2\phi)(K_s - K_b)}{(1 - \phi - K_b / K_s + \phi K_s / K_f)} + \frac{4G}{3} \quad (4)$$

$$Q = \frac{(1 - \phi - K_b / K_s)\phi K_s}{1 - \phi - K_b / K_s + \phi K_s / K_f} \quad (5)$$

and

$$R = \frac{K_s \phi^2}{(1 - \phi - K_b / K_s + \phi K_s / K_f)} \quad (6)$$

The poroelastic densities, ρ_{11} , ρ_{22} , and ρ_{12} are defined by

$$\rho_{11} + \rho_{12} = (1 - \phi)\rho_s \quad (7)$$

$$\rho_{22} + \rho_{12} = \phi\rho_f \quad (8)$$

and

$$\rho_{12} = (1 - \alpha)\phi\rho_f \quad (9)$$

where ρ_s and ρ_f are densities of solid and fluid, respectively. α is the tortuosity of trabecular connectivity. The bulk moduli, K_b , K_s , and K_f are bulk moduli of bone, solid bone matrix, and fluid. The method to estimate these effective mechanical properties is illustrated in Yoon et al. [3,4].

RESULTS AND DISCUSSION

Figures 1 and 2 show the relationship between the fast and slow wave speed and the porosity and charge density. If no charge density q_f is involved in governing equations (1-3), identical results shown in Yoon et al. [3] can be obtained.

The charge density q_f is varied from 0 to 1 (z axis) and it slightly changes the fast and slow wave speeds. Although the variation is not significant, we can estimate the charge density q_f after measuring the porosity by using micro-CT and the fast and slow wave speed by using ultrasound.

The results obtained in this paper is identical to the analytical form of Biot theory described in Williams [5] when the charge density q_f is ignored.

This is the first study showing the possibility to detect the effect of streaming potential (or electrokinetic effect) experimentally, and we will perform the experiment as a future study.

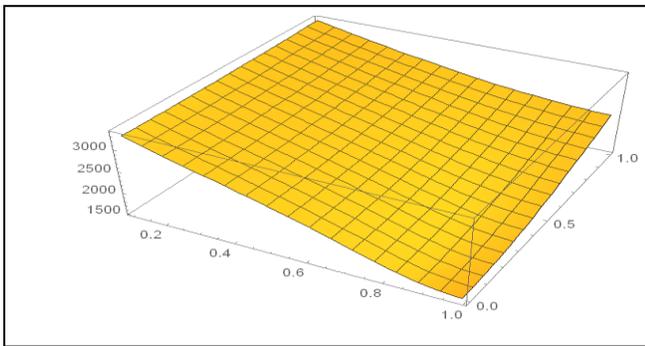


Figure 1: The fast wave speed (y axis) against porosity (x axis) and the charge density (z axis)

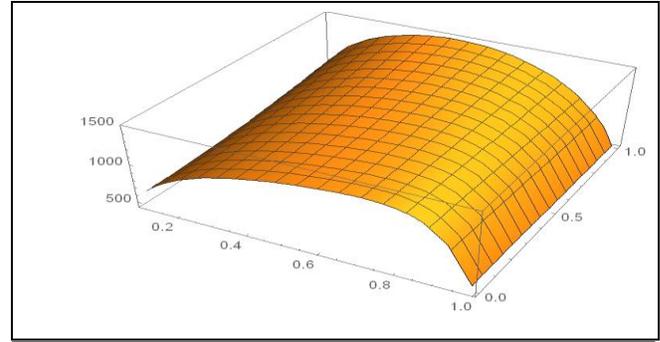


Figure 2: The slow wave speed (y axis) against porosity (x axis) and the charge density (z axis)

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