A MATHEMATICAL VENOUS MODEL

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INTRODUCTION

Mathematical models of the human circulatory system which can be simulated on digital computers are useful in evaluating the performance of total artificial hearts (TAHs) or ventricular assist devices (VADs). The major components of the human circulatory system can be modeled using differential equations, and these groups of differential equations can be combined to model the overall circulatory system. Progressive refinements can be made so that the complete set of differential equations models the system at a finer and finer scale. This paper presents two mathematical models of the venous system, developed for inclusion in a larger model of the human circulatory system, both of which are able to model the venae cavae collapse at low internal pressures.

THE VENOUS MODELS

Two venous models were developed. For a first approximation, the venous system was modeled using a single compliance element with a linear constitutive relation. Nonlinearities (such as collapse) are a consideration in the larger veins, but are negligible in the smaller vessels. The venous models described in this paper were designed to slot into an existing circulatory system model which ignores the right hand side (i.e. low pressure pulmonary loop) of the human circulatory system. Therefore, the pressures described in this paper are different than what might be found in the human circulatory system. In the aforementioned (pre-existing) circulatory system model the venous segment empties directly into the VAD. In the absence of a natural heart model, the venous system is soley reponsible for filling the VAD, requiring a venous pressure of $15mmHg$ as opposed to $5mmHg$.

SINGLE COMPLIANCE

As a first approximation the venous system can be modeled using a single compliance element whose constitutive relation is the volume response of the venous system to changes in internal pressure, as illustrated in Figure 1. This simplification (assuming a linear volume response to changes in pressure) is reasonable because a large portion of the venous system's total blood volume is contained in small diameter veins which would not be able to collapse or display other nonlinearities. Capillary response to changes in pressure is small.

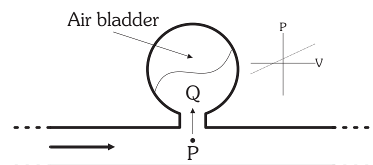

Figure 1: Schematic representation of the single compliance element venous model

PARAMETER SELECTION

The intercept and slope of the linear compliance element's consitutive relation can be derived from values found in literature. The intercept, or offset, is related to the nominal systemic volume and pressure, while the slope can be calculated by curve fitting data points using a least-squares linear fit.

In this work, the single compliance element was assumed to have a constant slope of $3500cm^5/N$, with an offset of $-0.4N/cm^2$ [5],[1], yielding an internal pressure of $18.75mmHg$ for the nominal blood volume of $2250ml$, and the resulting constitutive relation (1):

$$P_{vn}(V) = \Phi_c^{-1}(V) = \frac{V}{3500} - 0.4 \tag{1}$$

DUAL COMPLIANCE

The dual compliance model is based on the realization that nonlinear effects will be negligible in many of the peripheral vessels in the venous system. Only the larger venae cavae, which contain a relatively small percentage of the total venous blood volume, will exhibit noticeable nonlinearities. The venous system was modeled using two compliance elements. The first, linear, compliance element models the behavior of of the smaller peripheral veins which contain a large portion of the system's total blood volume. The second, nonlinear, compliance models the lumped behavior of the larger veins, which contain a smaller portion of the system's total blood volume.

Through the addition of a nonlinear resistance, the model can simulate the collapse of the larger veins at low internal pressures.

Two configurations of the dual compliance model were investigated. The first configuration (which is the main focus of this paper) was dubbed the "Serial Dual Compliance Model" (SDCM), while the second was referred to as the "Parallel Dual Compliance Model" (PDCM).

In the SDCM, the elements (two compliance and one resistance) are arranged sequentially in series. The linear compliance element is followed by the nonlinear resistance which is in turn followed by the nonlinear compliance element, as illustrated in Figure 2. Thus, the resistance element's constitutive relation depends on the pressure difference between the two venous segments, and the nominal venous volume is divided between the two compliances.

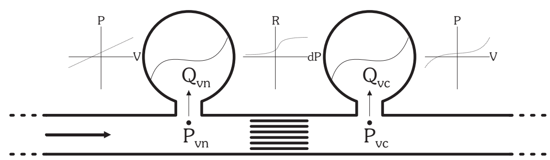

Figure 2: Schematic representation of the SDCM

In the parallel venous model, the linear compliance element is followed by a parallel arrangement of the nonlinear compliance element and the nonlinear resistance, as illustrated in Figure 3. The nonlinear resistance constitutive relation in this configuration is dependent on the flow into the nonlinear compliance, rather than the pressure difference between the two compliance elements.

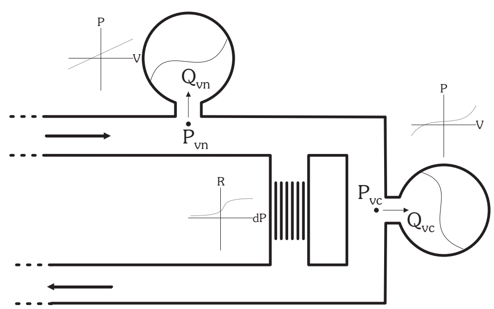

Figure 3: Schematic representation of the PDCM

PARAMETER SELECTION

In either dual compliance model, the constitutive relations for all three elements are interrelated. The linear compliance element is similar to that found in the single compliance model, with the pressure at nominal volume chosen based on literature, and the slope adjusted based on a least-squares linear fit. To help with system refinements, a piecewise function can be used to model the nonlinear compliance. Both the internal pressure at nominal volume and the internal pressure before collapse can be set based on values found in literature. The resistance function, which could also be modeled as piecewise linear, is chosen based on desired flow rate in the fully open and fully collapsed configurations, and the pressure drop across the entire venous segment.

In this work, the smaller, peripheral veins contain (nominally) $2L$ of the system's blood volume, and the venae cavae contain (nominally) $0.8L$ of the blood volume. For the linear compliance element, the pressure at nominal volume was chosen as $3.75mmHg$ (i.e., $0.05N/cm^2$), and the slope was matched to that of the single comliance model, giving a pressure of $-6.75mmHg$ (i.e., $-0.09N/cm^2$) at a volume of $2L$. This leads to Equation (2):

$$P_{vn}(V) = \Phi_c^{-1}(V) = \frac{V}{3500} - 0.6643 \qquad (2)$$

In the initial model, a piecewise constitutive relation was used for the nonlinear compliance. The nominal pressure was set to $3mmHg$ (i.e., $0.04N/cm^2$) at a volume of $800ml$, and the internal pressure before collapse was set to $0.75mmHg$ (i.e., $0.01N/cm^2$). The collapse volume was set to $200ml$, resulting in Equation (3):

$$\begin{aligned} P_{vc}(V) &= \Phi c^{-1}(V) \\ &= \begin{cases} 0.01 - (200 - V) \cdot 20000 & V < 200 \\ 0.01 + \frac{V-200}{20000} & 200 \leq V \leq 800 \\ 0.04 + (V - 800) \cdot 20000 & 800 < V \end{cases} \end{aligned} \qquad (3)$$

To prevent integration problems, small fillets were added to the piecewise function at each of the transition points.

As mentioned earlier, the venous pressures for both segments are higher than those typically found in literature [1],[3], but this is a necessary by-product our overall circulatory system model (which is not discussed here).

The resistance function was chosen to allow a nominal flowrate of $50ml/s$ when the vessel was collapsed and $400ml/s$ when it was fully open. For the SDCM, the offset, or transition pressure difference, was chosen as $1.5mmHg$ (i.e., $0.02N/cm^2$), based on the two compliance functions.

$$R_{vn}(\Delta P) = A + \frac{B}{1 + e^{-2 \cdot C \cdot ((\Delta P) - D)}} \qquad (4)$$

where:

A	is the minimum flow resistance
B	is the maximum flow resistance
C	affects the transition slope
D	controls the pressure offset

The simulation equations for the SDCM are then:

$$\dot{V}_{vn} = S_{f1} - \frac{\Delta P}{R_{vn}(\Delta P)} \quad (5)$$

$$\dot{V}_{vc} = \frac{\Delta P}{R_{vn}(\Delta P)} - S_{f2} \quad (6)$$

Equations (5) and (6) can be integrated with respect to time to predict the venous system behaviour.

RESULTS AND DISCUSSION

To test the venous collapse, Equations (5) and (6) were adapted slightly. The outflow source (S_{f2}) was replaced by a pressure sink, and a constant flow resistance was added to the outflow port. This modification is not necessary when the venous segment is inserted into the main circulatory system, but it *is* necessary when the venous segment is being simulated on its own. To produce Figures 4 and 5, the inlet volume flow rate and outlet pressure were varied. From $t = 0s$ to $t = 15s$, the inflow and outflow are $45ml/s$ and $-24mmHg$ (respectively). At $t = 15s$, these flow conditions are changed to $90ml/s$ and $-1mmHg$, giving a net inflow. At $t = 40s$, the outlet pressure was lowered again, to $-0.01mmHg$, increasing the rate of accumulation in the peripheral veins.

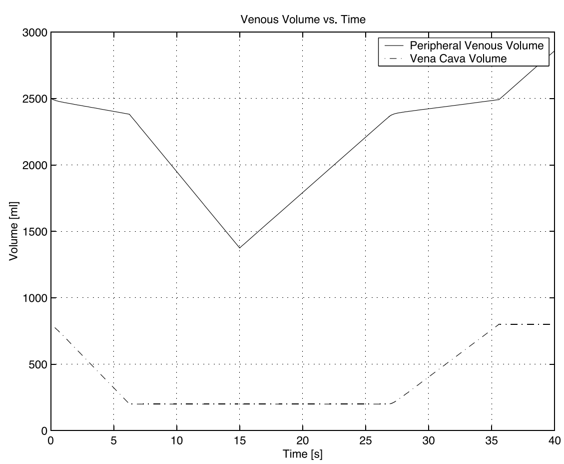

Figure 4: Venous volume with respect to time (illustrating venous collapse)

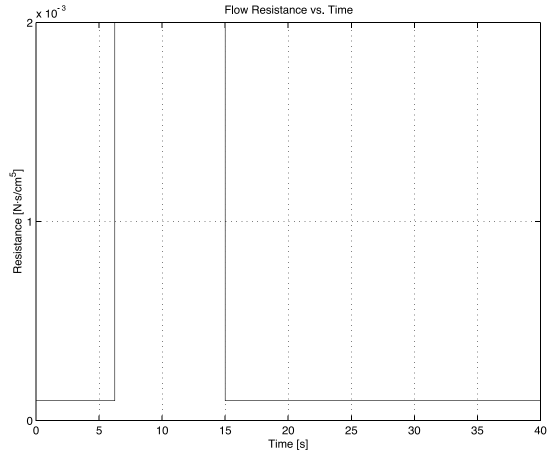

Figure 5: Venous flow resistance with respect to time (illustrating venous collapse)

The behaviour of the venous system is as expected; initially the two segments empty together, but the nonlinear compliance, containing a smaller portion of the total blood volume, reaches a critically low pressure at approximately $t = 6s$. The venous collapse is accompanied by a corresponding increase in flow resistance, visible in Figure 5.

When the new venous system is inserted into the main circulatory system model [5], its most profound effect is the reduction in downstream pressure oscillations, illustrated in Figure 6.

CONCLUSIONS

The dual compliance model is able to effectively model venous collapse at low internal pressures. It also reduces the downstream pressure oscillations when inserted in the main circulatory system model, a very positive effect.

REFERENCES

[1] Altman, P.L., Dittmer, D.S. (eds.) 1971. *Respiration and Circulation.* Bethesda, MD: Federation of American Societies for Experimental Biology.

[2] Conlon, M.J. 2000. *Design and Evaluation of a Neural Network-Based Controller for an Artificial Heart.* Carleton University, Ottawa, ON.

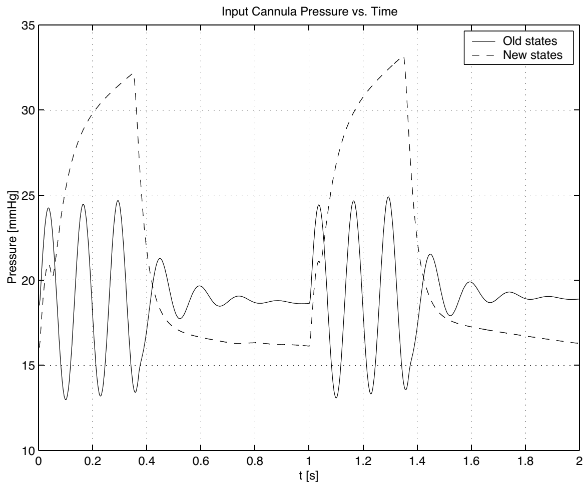

Figure 6: Downstream pressure oscillations in circulatory system model

[3] Croston, R.C., et al. 1973. *Computer Model of Cardiovascular Control System Responses to Exercise.* J. of Dynamic Systems, Measurement, and Control. **95:**301-307.

[4] Katz, A.I., et al. 1969. *Flow Through a Collapsible Tube.* Biophysical Journal. **9:**1261-1279.

[5] Menon, V. 1997. *Fuzzy Logic Controller for an Artificial Heart.* Carleton University, Ottawa, ON.